INFORMATION ASYMMETRY BETWEEN PRINCIPAL AND AGENT IN SOME PERFORMANCE EVALUATION MODELS

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ABSTRACT

The research question on problems that involves information asymmetry has been drawing more and more attention since the past decades, and in particular, two of the pioneers Bengt Holmström and Oliver Hart) in this field won the Nobel Prize of Economics in 2016. With the emergence of information economics, accounting researchers started focusing on the information asymmetry problems, with a particular interest and emphasis on moral hazard problems, within the firm. In this essay, we intend to fill the blank in this area by investigating some specific information asymmetry problems in managerial accounting under the presence of both moral hazard and adverse selection, or moral hazard and post-contract information asymmetry, respectively.

The first study analyzes the expected value of information about an agent's type in the presence of moral hazard and adverse selection. The value of the information decreases in the variability of output and the agent's risk aversion, two factors that are typically associated with the severity of the moral hazard problem. However, the value of the information about agent type first increases but ultimately decreases in the severity of adverse selection.

The second study draws attention to the tradeoffs associated with relying on pre-contracting ability measures in the design of executive compensation schemes. We show that the more sensitive of the ability signal to ability the more weight should be placed optimally, and the more precise of the ability signal the more weight should be placed optimally, in accordance with the informativeness principal. We further prove that under a broad class of distributions a linear aggregation of multiple pieces of pre-contracting information is sufficient for contracting purposes without loss of generality.

The third study investigates three mechanisms of organizational control: outcome control (contracting on the outcome), effort control (contracting on the signal of action), and clan control (employing an agent whose preferences are partially aligned with the principal's goal through a socialization process). In doing so, we expand the standard agency framework by introducing the concept of other-regarding preference and clan control to provide new insights into organizational control design.

I dedicate this dissertation to my wife and my parents, who continuously support me in my study.

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CHAPTER 1

INTRODUCTION

The research question on problems that involves information asymmetry have been drawing more and more attention since the past decades, and in particular, two of the pioneers Bengt Holmström and Oliver Hart) in this field won the Nobel Prize of Economics in 2016. Not long ago, firm was still viewed as a “black-box” that take inputs in and produces outputs in neoclassical economic theories. With the emergence of information economics, accounting researchers started focusing on the information asymmetry problems, with a particular interest and emphasis on moral hazard problems, within the firm. However, numerous studies have shown that many results and predictions drawn from pure moral hazard models are inconsistent with conclusions drawn from empirical research. As a consequence, many researchers call for theoretical work that considers other type of information asymmetry problems besides pure moral hazard problem. In this dissertation, we intend to fill the blank in this area by investigating some specific problems in accounting under the presence of both moral hazard and adverse selection, or moral hazard and post-contract information asymmetry, respectively.

A moral hazard problem may emerge if a principal cannot observe her agent’s action, such as his effort level. The principal can address this problem by making the manager’s compensation a function of some observable outcome that depends on the manager’s effort (typically referred to as pay-performance sensitivity or PPS). An adverse selection problem may arise when some relevant traits of the agent cannot be observed by the principal (e.g., the manager’s ability) so that the agent can misrepresent himself. The principal can address this type of problem by offering a menu of contracts, each of which would only be chosen by an agent of a particular level of ability. Pre-decision information asymmetry problem may occur when the agent acquires private information on the production or cost characteristics after the contract is agreed upon but before he makes any decision on his action. The principal can address this type of problem by delegating the decision choice to the agent instead of dictating the agent's choice of effort level.

In this essay, we present our findings using three studies. The first study analyzes the expected value of information about an agent's type in the presence of moral hazard and adverse selection. Information about the agent's type enables the principal to sort/screen agents of different types. The value of the information decreases in the variability of output and the agent's risk aversion, two factors that are typically associated with the severity of the moral hazard problem. However, the value of the information about agent type first increases but ultimately decreases in the severity of adverse selection. The decrease comes about because the means available to the principal to induce effort—namely, the pay–performance sensitivity—must also be used to sort/screen agents, and these two goals conflict. This decline in value occurs despite the monotonically increasing importance of the information in determining the principal's expected profits. Further, we show that the peak value of information occurs at a predictable level of adverse selection. These results imply that over some range, the importance of the information will be increasing, and the value of the information will be simultaneously decreasing, in the severity of adverse selection.

This second study draws attention to the tradeoffs associated with relying on pre-contracting ability measures in the design of executive compensation schemes. We recognized that, in designing the compensation scheme of an executive, the firm has access to noisy measures of the agent's ability pre-contracting. We formalized this intuition by studying a model in which the principal implements a contract contingent not only on the outcome of interest to the principal but also a noisy signal of the agent's ability. We show that more weight is placed on the ability signal optimally when the signal is more precise and has higher sensitivity to ability, in accordance with the informativeness principle. We further prove that under a broad class of distributions the principal can linearly aggregate multiple pieces of information without loss of generality.

The third study proposes an analytical model that integrates two parallel streams of literature that seek to identify optimal organizational design: market-based agency theory and organizational control theory. We study three mechanisms of organizational control: outcome control (contracting on the outcome), effort control (contracting on the signal on action), and clan control (employing an agent whose preferences are partially aligned with the principal's goal through a socialization process). In doing so, we expand the standard agency framework by introducing the concept of other-regarding preference and clan control to provide new insights into organizational control design. By rigorously defining different types of measurements faced by the principal, we are able to identify conditions under which outcome control, effort control, or clan control is optimal. We show that three forms of measurements—outcome measurement, effort measurement, task programmability—and socialization cost jointly determine the optimal control mechanism. Finally, we conduct a Monte-Carlo simulation to illustrate the analytical results. Overall, by integrating various features important in organizational control, agency theory, and behavioral economics, we sharpen the insights from earlier organizational control research and gain new insights on the design of optimal management control mechanism.

The remainder of this essay is organized as follows. In Chapter 2, we present a study titled "The value of pre-contract information about an agent's ability in the presence of moral hazard and adverse selection." In Chapter 3, we present a study titled "Relative weights on signals of ability versus outcome in the presence of moral hazard and adverse selection." In Chapter 4, we present a study titled "Optimal management control mechanism."

CHAPTER 2

THE VALUE OF PRE-CONTRACT INFORMATION ABOUT AN AGENT'S ABILITY IN THE PRESENCE OF MORAL HAZARD AND ADVERSE SELECTION

2.1 Introduction

In the process of hiring a new manager, a principal typically acquires, before contract negotiation, a variety of information about the candidate manager's ability.[1] Information may be hard or soft. Hard information, such as the candidate's educational background or past performance at prior positions is quantifiable and verifiable. Soft information, such as the candidate's reputation, may not be easily quantifiable nor verifiable. Yet in either case the compensation contracts offered by the firm may incorporate such acquired pre-contract information. If so, contracts can differ based not only on post-contract, future outcomes, but also on the pre-contract information. For example, contingent compensation (such as a bonus) may depend on realized outcomes, but also on the information known about the manager at the time of contract negotiation. Needless to say, the fixed compensation (such as a salary) can only depend on the pre-contract information.

In this paper, we develop a model to analyze the value of pre-contract information in a principal-agent setting with both moral hazard and adverse selection. A moral hazard problem may emerge if a principal cannot observe her agent's action, such as his effort level. The principal can address this problem by making the manager's compensation a

[1] We model "information" as simply anything that could improve the principal's knowledge about the agent's managerial ability.

function of some observable outcome that depends on the manager's effort (typically referred to as pay-performance sensitivity or PPS). An adverse selection problem may arise when some relevant traits of the agent cannot be observed by the principal (e.g., the manager's ability) so that the agent can misrepresent himself. The principal can address this type of problem by offering a menu of contracts, each of which would only be chosen by an agent of a particular level of ability.

In designing a menu of contracts under asymmetric information, we show that the principal faces a tradeoff in structuring incentive contracts to simultaneously deal with both moral hazard and adverse selection. When adverse selection is overlaid on top of moral hazard, we find that the best option available to the principal is to *modify* the compensation structure that would have been offered if the principal faced with only a moral hazard problem. The result is a weaker link between pay and outcome in some of the contracts, which decreases the incentive of the agent to exert effort. But if the principal did not modify the contracts in this way, the agent might agree to an even worse (from the principal's perspective) contracting outcome. Replacing an under-performing CEO with a new CEO, for example, can be extremely costly (see Taylor 2010). Thus, information about an agent's ability is fundamentally valuable because it allows the principal a freer hand to focus the compensation structure on mitigating the moral hazard problem.[2]

By analyzing a model with both moral hazard and adverse selection, we first derive the *ex ante* value of pre-contracting information about the agent's ability. We

[2] By "ability," we mean any trait of the agent such as intelligence, managerial talent, fit for the position, or other personal characteristics that will, for a given level of managerial effort, positively influence the expected outcome of interest to the principal. We also refer to the realization of this trait for a particular agent as the agent's "type."

show that, because of the dual purpose served by the pay-performance sensitivity (PPS) in addressing both the adverse selection and moral hazard problems, the value of the pre-contracting information on ability depends, interestingly, on features related to the moral hazard problem: the value decreases in the noisiness of the measure of effort and the risk-aversion of the agent. This is because the PPS in the menu of contracts, which would have been used strictly to address the principal's uncertainty regarding the agent's post-contracting action in a pure moral hazard setting, are now also the primary tools available to the principal to sort/screen the agents by ability.[3] The principal achieves sorting by distorting the PPS in the menu of contracts from the optimal value in a strictly moral hazard setting.[4] Thus, better pre-contract sorting comes at the expense of providing suboptimal post-contract incentives to the agent.

Because of this tension between pre-contract sorting and post-contract incentivization, the principal gains from better pre-contract information. If the principal becomes more convinced that the agent is of a particular type, the expected cost of modifying contracts on the menu decreases. In particular, the informed principal can gain by modifying contracts for other types in such a way that they become more costly to the principal if implemented but relatively more attractive to the types for which they are

[3] While "sorting" and "screening" are often used interchangeably in the literature, we use sorting hereafter. We examine a setting in which the principal designs a mechanism so that agents sort themselves. Eeckhout and Kircher (2010) refer to the mechanism as ex ante sorting as opposed ex post screening, in which, for example, buyers reveal their reservation values in an auction setting.

[4] Although in our model, the principal moves first by designing and offering a menu of contracts to the agent, it is possible to have the agent move first by signaling his type at some cost as in Spence (1973). Such signals may be interpreted as a special case of the information modeled in our paper so long as they are fixed by the time of contract negotiation.

intended. This is because it is less likely that any of these other contracts will ever take effect (because it is unlikely that the agent is of any of the types for whom these other contracts are intended). This modification allows the principal to make a corresponding beneficial modification (from her point of view) to the contract that is more likely to be implemented. Thus, the value of the pre-contracting information on ability is increasing in its precision and in its sensitivity to the agent's ability, because these tend to make the principal's beliefs about the candidate agent's ability more accurate.

Second, we show that the value of information is quasi-concave (i.e., single-peaked) in the dispersion of agent ability and the marginal productivity of ability (i.e., sensitivity of output to ability) relative to effort—two measures of the severity of the adverse selection problem—so that the value initially increases, but eventually decreases in these measures. This decline occurs even though expected output may be increasing in the severity of the adverse selection problem, in contrast to the decline in value brought on by an increase in the severity of the moral hazard problem, the decline of which is always accompanied by a decrease in output. This result arises because the principal faces a fundamental limit on the degree to which she can distort the contracts she offers— either because she cannot offer (or will not benefit from offering) a contract with a negative PPS, or because further distortion in the contract would make the agent unprofitable to employ. For those realizations of the information that would prompt an informed principal to offer the same PPS, the information does not have any value *ex post* because the contract is maximally distorted and cannot be further modified. This situation emerges more frequently as the adverse selection problem becomes more severe, causing the value of information to decrease in the severity of the adverse selection problem.

Additionally, we show that the peak value of information occurs at a level of adverse selection that is closely linked to the limit on the contract distortion. A specific set of parameter values determines the peak value, and the parameter space over which the value increases depends predictably on the features of the environment. However, we show that despite the decrease in value, the importance of the information in determining the principal's expected profits is strictly increasing in the severity of adverse selection.

Third, coupled with our result about the quasi-concavity of the value of information, this result on importance implies that there will be a range over which the value of information decreases in the severity of adverse selection while the importance of the information in determining the expected profit increases in that severity. In a nutshell, the intuition for this result is as follows: despite its increased importance *per se* in determining expected profits, the information becomes more costly for the principal to utilize when the adverse selection problem becomes too severe.

Our study builds on a large body of earlier work on how to resolve conflicts in the principal agent relationship, which arise due to the separation of ownership and management (Jensen and Meckling 1976; Fama 1980). Agency theory addresses the problems of information asymmetries (e.g., unobservable effort and agent type) between principals (owners) and agents (managers) by designing incentive contracts that help align their interests (Jensen and Zimmerman 1985; Eisenhardt 1989). In moral hazard settings, performance-based compensation enhances goal congruence, motivating agents to work hard so as to increase the payoff of the principal (e.g., Holmström 1979; Banker and Datar 1989; Bushman and Indjejikian 1993). Contemporaneous performance measures are useful because they provide information about the agent's unobservable effort (e.g., Holmström 1979; Banker and Datar 1989; Feltham and Xie 1994). For example, Holmström (1979) proposes that the principal can increase her expected payoff by explicitly including various performance measures in the contract, if the sufficient statistic condition is violated.

A large literature has also examined the role of asymmetric information about the manager's ability in an adverse selection setting (e.g., Maskin and Riley 1984; Rose and Shepard 1997). When agents possess different levels of ability that are unobservable to the principal, compensation schemes can be designed to sort agents with inferior ability or induce agents to self-select contracts that reveal their ability. Without judicious sorting, contracts aimed at one type of manager will also be attractive to other types (Stiglitz 1977; Lazear 1986). Adverse selection theory derives mechanisms that can effectively sort agents on the basis of their unobservable ability (Harris and Raviv 1978; Rothschild and Stiglitz 1976; Salop and Salop 1976). Agents with higher ability are induced to choose steeper bonus contracts (Darrough and Melumad 1995) or stock options (Arya and Mittendorf 2005). For effective sorting, better agents may have to face more risk. The principal trades off the benefit from sorting against the cost of sorting. For example, Darrough and Melumad (1995) use a two-type pure adverse selection model to examine circumstances under which it is optimal to maximize divisional or short-term objective rather than firm-wide or longterm objectives. In some cases, the optimal contract is designed to attract the most talented managers, but in others, the optimal contract pools different types of managers because sorting is too costly. Empirical studies (Rose and Shepard 1997; Gabaix and Landier 2008; and Terviö 2008) show that executives are paid more in firms that are large and heavily diversified because of matching between high-ability CEOs and firms that are difficult to manage. Henderson and Fredrickson (1996) find that executive compensation is positively related to information-processing ability because "the ability to cope with large volumes of diverse information is likely to be both rare and critical to organizational performance."

A number of studies examine principal-agent relationships in the presence of both moral hazard and adverse selection. Goldmanis and Ray (2014) focus on deriving optimal linear contracts to sort agents such that agents with ability below the optimal threshold would not join the firm. Dutta (2008) characterizes optimal contracts with agents who are endowed with different degrees of general or firm-specific expertise that affect their reservation utility. He shows that optimal PPS depends on the specificity of managerial expertise. Lazear (1986) examines in a dichotomous compensation scheme setting (hourly wage and piece rate) to resolve both moral hazard and adverse selection. Using a sample of nearly 3,000 workers in a manufacturing company, he finds support for his prediction that average output per worker and the average ability of workers increase after switching from a compensation scheme of hourly wages to piece rates. Banker, Darrough, Huang, and Plehn-Dujowich (2013) also study a setting with moral hazard and adverse selection where the principal has access to information about ability—the past performance of a firm under a candidate's management—in a dynamic setting and show how a sequence of information influences the optimal menu of present and future contracts.

Optimal contracting and information aggregation have also been studied in the literature on career concerns in multi-period settings. For example, Gibbons and Murphy (1992) show that the optimal "explicit incentives" become stronger as agents approach their retirement and their career concerns diminish. Noting that there are multiple performance measures from various periods, the question whether to aggregate or disaggregate performance measures is addressed by Autrey, Dikolli, and Newman (2010) and Arya and Mittendorf (2011), among others. Disaggregated measures may not be always optimal.

Finally, the literature on pre-decision information centers on situations in which the agent gains private information about the production process after the contract is signed and explores how the principal can create incentives to induce the agent to use his private information to the principal's benefit. Under certain conditions, such information increases the principal's welfare by improving the coordination problem between the principal and the agent. In examining pre-decision information, Baiman and Sivaramakrishnan (1991) explore when it is optimal to endow private information to the agent. Kim and Suh (1991) develop a framework to optimally determine how much information to gather about an agent's action in a strictly moral hazard setting.

Our paper contributes to the literature by focusing on the value of pre-contract information on ability in a setting with both moral hazard and adverse selection. In our model, the principal's optimal contract is calibrated after the principal observes a signal(s) of the agent's ability but before a menu of contract is offered and accepted. The setting with pre-contract information enables us to examine the role and the value of information obtained by the principal about a particular agent prior to designing a compensation scheme. In practice, such information is routinely gathered during the search for an agent as well as in many other analogous situations, underscoring the importance of this type of information.[5]

The remainder of the paper is organized as follows. In section 2.2, we develop the agency model, and in section 2.3, we derive the optimal contracting mechanism and present some results that arise as an immediate consequence. In section 2.4, we define the concept of the value of information and derive the expression characterizing that value, followed by a discussion on the relationship between information and profits. We conclude with section 2.5.

2.2 The Agency Model

A risk-neutral principal (owner) wishes to contract with a risk- and effort-averse agent (manager) to operate her firm. After a contract is agreed upon, the agent exerts unobservable effort e, which represents any action undertaken by the agent on behalf of the principal. The agent is endowed with ability $a \in \{H,L\}$, with $H > L$, the true value of which is known only to the agent.[6]

Thus, the principal faces a moral hazard problem (with respect to effort e) and an adverse selection problem (with respect to managerial ability a). The principal's rational (that is, correct) prior belief (i.e., pre-contract information) about the agent's ability is $P(a = H) = v$ and $P(a = L) = 1 - v$.

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[5] In Dutta (2008), the agent's reservation utility depends on his ability whereas in our model the reservation wage is assumed constant across agent types.

[6] We refer to agents of the two different levels of ability as "types." While agents know their type, we assume in our paper that they are not able to credibly signal their type, or equivalently that any such signaling is already incorporated in the principal's prior beliefs.

We adopt the standard framework that supposes a linear contract, exponential utility on the part of the agent, and normal disturbance to firm output (see Holmström and Milgrom 1987; Bose, Pal, and Sappington 2011). Our main results regarding the value of information, however, do not depend on these assumptions. An agent endowed with ability a who exerts effort e generates the outcome:[7]

$$\tilde{y}(e, a) = \gamma_a a + \gamma_e (e + \epsilon).$$

Dividing the equation by γ_e yields

$$y(e, a) = \frac{\tilde{y}(e, a)}{\gamma_e} = \frac{\gamma_a}{\gamma_e} a + e + \epsilon$$

$$= \lambda a + e + \epsilon, \qquad (2.1)$$

where $\frac{\gamma_a}{\gamma_e}$ is relabeled as λ and ϵ is a mean-zero normal disturbance term with variance σ_ϵ^2. $\lambda \geq 0$ represents the productivity of ability (or the sensitivity of the outcome to the agent's ability), and the productivity of effort (the sensitivity of the outcome to the agent's effort) is normalized as one. Because ability is not directly observable by the principal, the parameter λ also relates to the importance of the agent's private information in the principal-agent relationship (that is, to the importance of the adverse selection problem). Note that the outcome function is separable in ability and effort. This implies, for example, that the productivity of effort is independent of ability or agent's type.[8]

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[7] We adopt this formulation, where the disturbance is multiplied by the marginal productivity of effort, so that in our modified production function, we can change the relative productivity parameter without affecting the noisiness of the output. In the alternate specification, changes in the marginal productivity of effort affect the value of information in part by changing the informativeness of the realized output regarding effort, something that is not of direct interest.

[8] The separability assumption is not as restrictive as it may appear. Suppose that the outcome is Cobb-Douglas, $y(e,a,z) = a^{\gamma_a}(e + \varepsilon)^{\gamma_e}$. Taking logs, this becomes $\ln y(e,a,z) = \gamma_a \ln a + \gamma_e \ln(e + \varepsilon)$. Hence, if we redefine the outcome, effort, and ability in log terms,

The principal designs a menu of compensation contracts contingent on the outcome y that induces the agent, irrespective of his ability, to exert the desired level of effort (incentive compatibility), truthfully reveal his ability (truth-telling), and voluntarily sign the contract (individual rationality). The principal adopts compensation contracts of the form:

$$w_a(y) = \alpha_a + \beta_a y \text{ for } a \in \{H, L\}, \tag{2.2}$$

where α_a represents the salary intended for an agent of type a and β_a represents the PPS for the same agent type. The objective of the principal is to maximize the expected outcome net of the agent's compensation. Although we suppress them here, the values of y, α_a, and β_a, in general, will also be functions of other parameters of the problem.

The principal may be able to observe some information about the agent's ability prior to negotiating the contract. The conditional distribution of the information $\mathbf{z} \in \mathbf{Z}$ given the agent's true ability a is denoted $g(\mathbf{z}|a)$ for each $a \in \{H,L\}$, where $\mathbf{z}$ is a vector of n pieces of component information $(z_1, z_2, \cdots, z_n)$. We first assume that z is a scalar for simplicity and then address the issue of multiple signals in Section 5. We also use the notation $g_a(z) \equiv g(z|a)$ for the remainder of the paper, and similarly $G_a(z) \equiv G(z|a)$ for the corresponding cumulative distribution function. If she observes information, the principal updates her belief about the distribution of the agent's ability by using Bayes' rule.[9] Thus,

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then we obtain a linear production function. The only difference is in the interpretation of the parameters: in the Cobb-Douglas case, γ_e represents the elasticity of the outcome with respect to effort, while in the linear case, it represents the sensitivity of the outcome to effort.

[9] Milbourn (2003) also studies a setting in which the principal updates her belief about agent's ability upon observing information.

for a realized value of z, the density function $f(a|z)$ representing the principal's posterior belief is in general given by

$$f(a|z) = \frac{g_a(z)f(a)}{\int_\Omega g_a(z)f(a)da}.$$

In our discrete ability setting, $\Omega = \{H,L\}$, this becomes

$$P(a|z) = \frac{g_a(z)P(a)}{vg_H(z) + (1-v)g_L(z)}. \tag{2.3}$$

where $P(H) = v>0$ and $P(L) = 1 - v>0$ are the principal's prior beliefs. This implies that

$$\frac{u}{1-u} = \frac{v}{1-v}\frac{g_H(z)}{g_L(z)} = \frac{v}{1-v}LR(z), \tag{2.4}$$

where $u(z) \equiv P(a{=}H|z)$ and $1{-}u(z) = P(a{=}L|z)$ represent the principal's posterior beliefs, and $LR(z)$ is the likelihood ratio.[10]

The probability density function $g_a(z)$ represents the frequency of observing information z when the agent is type a. Thus, the ratio $\frac{g_H(z)}{g_L(z)} = LR(z)$ is the relative likelihood of observing information z when the agent is of type H compared to when the agent is of type L. The principal updates her prior on this ratio according to Bayes' rule by multiplying it by the relative likelihood as shown in equation (2.4). We make the standard assumption below regarding the likelihood ratio (Milgrom 1981), which we maintain throughout:

Assumption 2.1 (Monotone Likelihood Ratio Property for Information) *The likelihood ratio $LR(z)$ is increasing in z.*

The agent's preferences exhibit constant absolute risk aversion (CARA) with coefficient R (see Dutta 2008, Datar, Kulp, and Lambert 2001, Holmström and Milgrom

[10] For the rest of the paper, we suppress the argument z in the function u(z).

1987, and Feltham and Xie 1994). The agent exerting effort e incurs a dollar-equivalent cost $C(e) = ce^2/2$.[11] As such, the certainty equivalent value of the contract specifying the pair (α,β) designed for an agent of type a consists of a salary and bonus minus the costs of risk and effort:

$$CE(\alpha,\beta) = \alpha_a + \beta_a(\lambda a + e) - \frac{R\beta_a^2\sigma_y^2}{2} - \frac{ce^2}{2}, a \in \{H, L\}. \tag{2.5}$$

We denote the agent's reservation utility as r_0.[12] We discuss some implications of relaxing the assumption that both types have the same reservation utility below. We maintain the assumption that $r_0 < \lambda L$, which ensures that the principal will find it profitable to employ the type L agent even when he exerts zero effort. Relaxing this assumption complicates the analytical results without changing them qualitatively.[13]

The timing of events in this model is as follows. First, the principal may observe information about the agent's ability, in which case she updates her beliefs accordingly. She then designs the compensation mechanism, which consists of a menu of contracts containing two contracts each of which is intended for a particular agent type.[14] After

[11] As in Dutta (2008) and Feltham and Xie (1994), the quadratic cost function assumption is made for convenience. All results continue to hold for a general cost function that is increasing and convex.

[12] Dutta (2008) relaxes the assumption that agent type and reservation utility are independent.

[13] In particular, relaxing this assumption leads to a range of beliefs over which the principal will exclude the L type agent and will offer the H type agent the pure moral hazard contract. The principal's profits in this scenario closely mimic those in the case we study here.

[14] By appealing to the revelation principle, we solve for a truth-inducing menu of contract. Compensation mechanisms in practice may not be a direct mechanism and may involve non-truthful reporting. There are alternative optimal mechanisms characterized

observing the menu, the agent decides what ability to report to the principal (which in equilibrium is his actual ability) and whether to accept the corresponding employment contract (which, in equilibrium, he does). The contract is signed, and the agent decides what level of effort to exert. Finally, the outcome is realized and is divided between the principle and the agent according to the terms of the contract.

2.3 The Principal's Mechanism Design Problem under Moral Hazard and Adverse Selection

The principal's problem is to maximize the expected outcome net of the agent's compensation, given her beliefs regarding his ability. Suppose that the principal believes the agent is of type *H* with probability *x*. For an uninformed principal, $x = v$, while for an informed principal, who acquires additional information about the agent, $x = u$, where v and u are prior and posterior beliefs about the type. Then the principal solves

$$\max_{\{\alpha_a,\beta_a,e(a)\}} \begin{array}{c} xE\big[y(e(H),H) - w\big(H, y(e(H),H)\big)\big] \\ +(1-x)E\big[y(e(L),L) - w\big(L, y(e(L),L)\big)\big] \end{array}, \tag{2.6}$$

subject to

$$e(a) = \arg\max_{\tilde{e}} CE(\alpha_a, \beta_a | a) \text{ for all } a \in \{H, L\}, \tag{2.7}$$

$$CE(\alpha_a, \beta_a | a) \geq r_0 \text{ for all } a \in \{H, L\}, \tag{2.8}$$

$$CE(\alpha_a, \beta_a | a) \geq CE(\alpha_{\tilde{a}}, \beta_{\tilde{a}} | a) \text{ for all } a, \tilde{a} \in \{H, L\}, \tag{2.9}$$

where $CE(\alpha_{\tilde{a}}, \beta_{\tilde{a}} | a)$ is the certainty equivalent of an agent of type a when claiming to be of type $\tilde{a}$ and optimizing accordingly. Equation (2.7) is the agent's incentive

by complex communication between the principal and agent, and in which the agent lies in equilibrium about his ability. See Ronen and Yaari (2001) who develop a model with an explicit nontruth-telling equilibrium. Indeed, it may be more realistic in some cases to imagine the process of interview and negotiation between an owner and a prospective manager to unfold in this way.

compatibility (IC) constraint, equation (2.8) is the individual rationality (IR) constraint, and equation (2.9) is his truth-telling (TT) constraint. We also impose $e(a) \geq 0$ for $a \in \{H,L\}$. Note that the principal is willing to hire either a type H or type L agent. Her mechanism design problem is to derive a menu of contracts designed judiciously for each type.

We first provide the solution to the principal's mechanism design problem under both moral hazard and adverse selection in the following lemma. We then discuss two special cases: pure moral hazard and pure adverse selection.

Lemma 2.1 (Solution to the Principal's Problem) *Suppose the principal rationally believes that the agent is of type H with probability x, where $x \in \{v,u\}$ and $0<x<1$. Denote $A \equiv c\lambda(H - L)$. Further, define $\bar{x} \equiv \frac{1}{A+1}$. Then the optimal pay-performance sensitivity (PPS) for the H type agent is*

$$\beta_H = \frac{1}{cR\sigma_y^2 + 1}, \tag{2.10}$$

and the optimal PPS for the L type agent is

$$\beta_L = \begin{cases} \dfrac{1 - A\frac{x}{1-x}}{cR\sigma_y^2 + 1} & if\, 0 < x < \bar{x} \\ 0 & if\ \bar{x} \leq x < 1 \end{cases}, \tag{2.11}$$

The expected total compensation of the H type agent is

$$E[w_H] = \begin{cases} r_0 + \dfrac{1}{2c\left(cR\sigma_y^2 + 1\right)} + \dfrac{2A\left(1 - A\frac{x}{1-x}\right)}{2c\left(cR\sigma_y^2 + 1\right)} & if\, 0 < x < \bar{x} \\ r_0 + \dfrac{1}{2c\left(cR\sigma_y^2 + 1\right)} & if\ \bar{x} \leq x < 1 \end{cases}, \tag{2.12}$$

and the expected total compensation of the L type agent is

$$E[w_L] = \begin{cases} r_0 + \dfrac{\left(1 - A\frac{x}{1-x}\right)^2}{2c\left(cR\sigma_y^2 + 1\right)} & if\, 0 < x < \bar{x} \\ r_0 & if\ \bar{x} \leq x < 1 \end{cases}, \tag{2.13}$$

The expected profit of the principal when she is matched with an H type agent is

$$
E[\pi_H] = \begin{cases} \lambda H - r_0 + \dfrac{1}{2c\left(cR\sigma_y^2+1\right)} - \dfrac{2A\left(1 - A\dfrac{x}{1-x}\right)}{2c\left(cR\sigma_y^2+1\right)} & if\, 0 < x < \bar{x} \\ \lambda H - r_0 + \dfrac{1}{2c\left(cR\sigma_y^2+1\right)} & if\ \bar{x} \leq x < 1 \end{cases}, \quad (2.14)
$$

and the expected profit of the principal when she is matched with the L type agent is

$$
E[\pi_L] = \begin{cases} \lambda L - r_0 + \dfrac{\left(1 - A\dfrac{x}{1-x}\right)^2}{2c\left(cR\sigma_y^2+1\right)} & if\, 0 < x < \bar{x} \\ \lambda L - r_0 & if\ \bar{x} \leq x < 1 \end{cases}, \quad (2.15)
$$

The expected profit of the principal over ability is then

$$
E_a[\pi] = \begin{cases} \lambda(xH + (1-x)L) - r_0 + \dfrac{x}{2c\left(cR\sigma_y^2+1\right)} \\ -x\dfrac{2A\left(1 - A\dfrac{x}{1-x}\right)}{2c\left(cR\sigma_y^2+1\right)} + (1-x)\dfrac{\left(1 - A\dfrac{x}{1-x}\right)^2}{2c\left(cR\sigma_y^2+1\right)} & if\, 0 < x < \bar{x} \\ \lambda(xH + (1-x)L) - r_0 + \dfrac{x}{2c\left(cR\sigma_y^2+1\right)} & if\, \bar{x} \leq x < 1 \end{cases}, \quad (2.16)
$$

Proof. Please see the Appendix.

In order to interpret the results of Lemma 2.1, consider first two special cases: (1) pure moral hazard (when there is no ability difference, or ability is observable), and (2) pure adverse selection (when output is deterministic given effort, or effort is observable). In the first case, if $H = L$ in our formulation, then $A = 0$, implying that $\bar{x} = 1$, so that the principal's beliefs about agent type do not matter. In this case, it is easy to see that the principal would offer a contingent contract to induce effort with $\beta_H = \beta_L = \frac{1}{cR\sigma_y^2+1}$. The principal will offer the same β in both contracts because effort productivity is assumed the same for both types of agents, and there is no tradeoff here between agent motivation and ability sorting. Furthermore, inspection of equation (2.16) shows that in this special case, the principal's expected profits are decreasing in R and in σ_y^2, validating these parameters as measures of the severity of the moral hazard problem.

In the case of pure adverse selection, with effort being observable, agents will be compensated for their effort expended regardless of the output level (or equivalently,

$\sigma_y^2 = 0$ so that output is deterministic given effort). In either case, the agent faces no risk. Not being able to determine the agent's type, however, the principal still faces a meaningful adverse selection problem. The results of Lemma 1 continue to apply in this special case, so that the principal would offer a menu of contracts, with $\beta_H = 1$ and $\beta_L = \max\left\{0, 1 - A\frac{x}{1-x}\right\}$, and the high-ability agent would earn information rent in excess of his reservation utility. And it can be shown by analysis of equation (16) (proof omitted) that the principal's expected profits are decreasing in λ and in $(H - L)$, validating these parameters as measures of the severity of the adverse selection problem.[15] Comparison of the pure moral hazard case and the more general case analyzed in Lemma 1 highlights the role of adverse selection in the principal's problem of designing optimal contracts.

It is immediate to see that $A = 0$ ($\lambda > 0$ and $H - L > 0$) captures the adverse selection problem. Therefore, as λ and/or $(H - L)$ increase in magnitude, the severity of the adverse selection problem also increases and the optimal contracts deviate further from the pure moral hazard case. The natural interpretation is that as the relative marginal productivity of ability becomes higher and/or the type differential between H and L becomes greater, the adverse selection problem becomes more important, and the PPS for the type L agent is distorted more, as clearly seen in equation (2.11). Of course, β_L cannot be negative, so when $x = \bar{x}$, $\beta_L = 0$ and remains at that level even if x increases further. In

[15] The use of a positive PPS to induce effort in this special case is due to our assumption of a linear contract. However, relaxation of this assumption does not qualitatively change the outcome. The principal may instead offer a pair of contracts, each specifying a wage and a required level of output that can only be met by exerting the proper amount of effort, which of course depends on the ability of the agent. In that case, the high-ability agent continues to put forth the first-best level of effort and to earn information rent; the low-ability agent exerts less than the first-best effort level and earns only his reservation utility; and the principal's expected profits are decreasing in λ and in $(H - L)$.

this case, a type L agent, if hired, would produce $E(y) = \lambda L$ and exert no effort. That $\beta \geq 0$ restricts the principal's ability to deal with adverse selection when its severity is high.

Note that the PPS for type H is determined by the cost of effort, risk aversion, and outcome variance. However, the presence of an adverse selection problem causes the principal to reduce the PPS for type L (when $x \leq \bar{x}$, which ensures that $\beta_L \geq 0$). In other words, the distortion to the type L agent's contract arises only because of the principal's attempt to sort agents by ability. The distortion is necessary in order to prevent the type H agent from opting for the L type contract; she must introduce some difference between the contracts that makes the L type contract less attractive to the type H agent. Because he knows that his output is likely to be higher (because of his ability) for a given effort level, the type H agent is more interested in an incentive-based contract than is the type L agent. Thus, lowering the PPS on the type L contract can induce the type H agent to choose the intended contract. And this distortion increases in the difference in ability between the type H and type L agents and in the relative marginal product of ability λ for $x \leq \bar{x}$. Thus, β_L decreases in x over this range ($x \leq \bar{x}$). This result, which is also standard, is due to the tradeoff faced by the principal: she balances the information rent that must be paid to the H type against the lower level of effort induced for the L type. However, as the principal becomes more convinced that the agent is of H type, the expected cost of distorting the L type agent's contract decreases (because it is unlikely to be implemented), so the optimal contract menu entails a more severe distortion in β_L relative to β_H. Again, this comes at a price: if the agent turns out to be of type L, he will exert less effort because of the lower incentive provided by the distorted PPS in his contract.

In sum, as is standard in adverse selection models, a distortion in PPS appears only in the contract for one type. The type H agent's action would be in all cases the same as in the case of pure moral hazard. However, β_L, the PPS offered to the L type agent, is distorted downward. In particular, β_L is strictly less than β_H, and this distortion is increasing in the difference in ability between the type H and type L agents and in the relative marginal product of ability λ for $x \leq \bar{x}$. However, because β_L cannot be negative, $\beta_L = 0$ when $x \geq \bar{x}$. Equation (2.12) shows that when $x \leq \bar{x}$, the type H agent receives an information rent in addition to the compensation for his effort and risk, while when $x \geq \bar{x}$, he receives only his reservation utility.[16]

The tradeoffs faced by the principal are illustrated by the expressions for expected profits in equations (2.14) and (2.15). The expected profit when the principal is faced with a type L agent, $E[\pi_L]$, decreases in $\lambda(H-L)$ (i.e., in the severity of the adverse selection problem) for a fixed value of output due to ability. This is because the distortion in the type L contract that is required to meet the truth-telling constraint increases in these objects, and this distortion necessarily lowers expected profit.

The effect of the adverse selection problem on expected profit when the principal faces an H type agent is more complicated. Consider the following decomposition of this expected profit from equation (14):

[16] Note that a positive perturbation in the reservation utility of the H type agent would have no effect on the optimal contracts. Because the H type receives strictly more than his reservation utility under the contract described above when $x \leq \bar{x}$, he would still accept the contract and would exert the same level of effort. And because the L type strictly prefers his contract to the H type contract, both truth-telling constraints will continue to hold. Only for discrete increases in the H type reservation wage—in particular, for increases that make the H type unwilling to accept the contract—will the optimal menu of contracts change.

$$E[\pi_H] = \lambda H - r_0 + \frac{1}{2c(cR\sigma_y^2 + 1)} - \frac{2A\left(1 - A\frac{x}{1-x}\right)}{2c(cR\sigma_y^2 + 1)}. \qquad (2.17)$$

For a fixed value of output due to ability, the adverse selection problem becomes more severe as $\lambda(H{-}L)$ increases (or as $A \equiv c\lambda(H - L)$ increases, given c), which affects the information rent paid to the H type. The information rent increases in $\lambda(H{-}L)$ when $\lambda(H{-}L) \approx 0$; that is, it increases in the severity of the adverse selection problem. However, the information rent peaks at some point because severely distorting the L type contract makes it increasingly unattractive to the H type, which ultimately lowers the required information rent. This causes the information rent to approach zero at a value of $\lambda(H{-}L)$ that corresponds to $x = \bar{x}$.

The value $\bar{x}$ also plays an important role in setting optimal contracts. The existence of a threshold of this type is not unique to our setting; Dutta (2008) shows that such a threshold arises in a more general setting of adverse selection and moral hazard. Once the belief threshold $\bar{x}$ is reached, the principal can no longer benefit from further distorting the L type contract because the associated PPS, and with it the effort of the L type, have already been pushed down to zero and cannot be further distorted. The information rent accruing to the H type also reaches zero at this point, so that the outcome is the same as the pure moral hazard case if the agent's type turns out to be H. If the agent's type is L, the principal settles for the output produced by the L type with zero effort. For values of x above this threshold, the contracts offered and the expected wages paid do not depend on x. The expected net profit continues to depend on x, but only because a higher x implies that the agent is more likely to be H type, and the expected output due to the agent's ability (which the principal captures in full) and the surplus from the agent's effort (which is positive only for the H type agent) are both higher for the H type than for the L type.

Finally, note that the optimal contracts and the principal's expected profit in equation (2.16) are straightforward functions of the principal's belief x regarding the agent's type. As a result, the above expressions hold whether $x = v$ is the principal's prior belief regarding the agent's type, or $x = u$ is the principal's posterior belief. An implication of this fact is that subsequently, we can use these expressions to examine the effects of information about the agent's type strictly by way of its effect on the principal's belief. We use this fact to derive the following two lemmas describing the relationship between the optimal compensation scheme and information about ability. Recall that the information about an agent's ability is denoted by z.

Lemma 2.2 (Negative Relationship between PPS and Information about Ability) *The optimal pay-performance sensitivity (PPS) for the L type agent is decreasing in the realized values of the information about ability:*

$$\frac{\partial \beta_L(z)}{\partial z} \leq 0. \tag{2.18}$$

Proof. The result follows from combining equations (2.11) and (2.4), and invoking Assumption 1.

Lemma 2.3 (Total Profit and Information about Ability) *The expected profit of the principal is increasing in the realized values of the information:*[17]

$$\frac{\partial E_a[\pi(z)]}{\partial z} \geq 0.$$

Proof. Please see the Appendix.

Information suggesting higher ability implies a greater chance of actually facing an H type agent. The greater distortion induced in the L type contract is more than offset by the reduction in information rent and the lower probability that the L type contract will

[17] Expectations are taken over the uncertainty in the outcome and the unobservable ability.

be accepted (because it is less likely that the agent is actually of *L* type). The net effect is an increase in the principal's expected net profit.

2.4 The Value of Information about Ability

In a setting with adverse selection and moral hazard, the principal faces a tradeoff in determining two factors in the menu of contracts. In order to successfully sort the two types, she trades off a distortion in the *L* type contract against paying information rent to the *H* type. The distortion in the *L* type contract involves lowering its PPS, which makes it less attractive to the *H* type, who expects to produce more due to his ability. This allows the principal to lower the information rent she pays in the *H* type contract, but at a cost. As the principal distorts the *L* type contract by making its PPS very low in order to induce the *H* type agent not to choose this contract, she faces the risk that the agent will in fact turn out to be of type *L*. In that case, the highly suboptimal *L* type contract would be selected. On the other hand, if the principal opts to keep the PPS in the *L* type contract near the PPS for the H type, then she can only deter the *H* type agent from defecting to the *L* type contract by loading the *H* type contract with a high information rent. If the agent in fact turns out to be of *H* type, then the principal would have to pay the high information rent.

The optimal balance between these two potential costs is influenced by the probability with which each cost will ultimately have to be paid. By virtue of the truth-telling constraint, these probabilities are equivalent to the probabilities that the principal actually faces each type of agent (i.e., the frequency of *H* and *L*). As a result, information that improves her knowledge regarding which type of agent she faces is valuable. Once information is observed, the principal updates her beliefs according to the conditional probabilities implied by the information, and the updated beliefs reflect more accurately the agent's actual type, on average. The principal can then calibrate the menu of contracts offered to reflect her updated beliefs and, on average, the resulting menu is better tailored to the agent type that she actually faces. She does this by further distorting the contract intended for the agent type that she is less likely to be facing, and by lowering the distortion of the contract aimed at the agent type that she is more likely to be facing.

As a result of the improvement in the menu of contracts offered, the expected net profit accruing to the principal increases, as we will show below. We define the value of the information as this change in expected profit.

2.4.1 General Properties of the Value of Information about Ability

We first outline the properties of the value of information for a general distribution of agent ability. We then derive properties of the value of information in our setting of binary agent type so as to use these results to study the determinants of the value of the information.

If the principal can observe information about ability (informed principal), then her expected profit is $E_a[\pi|z]$, while without information (uninformed principal), her expected profit is $E_a[\pi]$, where E_a denotes expectation over the ability distribution. The value of information z is then $E_a[\pi|z]-E_a[\pi]$ conditional on the realization z. Therefore, before z is realized, the expected profit over the unconditional ability distribution is

$$E[V(z)] = E_z\big[E_a(\pi|z) - E_a[\pi]\big], \tag{2.19}$$

where $E[V(z)]$ is the value of information.

The first fact to note is that, for any distributions of ability and information, preferences of the principal and the agent, and contract structure, the value of information is weakly positive. This is not to say that the principal's expected profits after observing the information are necessarily greater than her expected profits before observing it. Indeed, if the principal were to receive information that suggested that the agent's ability is very likely to be low, then her expected profits are likely to be lower than she had initially supposed. But this does not imply a negative value of information. In such a case, she would expect her profits to be low not because of the fact of having observed the information *per se*, but because the particular realization of the information will have revealed to her that the agent's ability is likely to be low.

The critical properties that determine the value of information turn out to be those of the conditional information distributions $g_a(z)$. To develop intuition, we first consider two extreme cases: distributions that will cause the information to attain its least and its greatest possible values. First, assume that $g_a(z) = h(z)$ for $a \in L, H$; in other words, the distribution of information z is independent of ability a. Clearly, z is entirely uninformative of a. Then, the value of the information is exactly zero. In such a case, the principal will choose not to observe it as long as the cost of doing so is strictly positive.

Next, consider the case when $g_a(z) = l(a)$, where $l(a)$ is a strictly monotonic function. In such a case, a given realization of the information will correspond perfectly with a particular agent's type; in other words, z will perfectly reveal the value of a. This perfect information represents the greatest possible value, which will be strictly positive so long as the production function nontrivially depends on the agent's ability. With this perfect information, the principal can effectively eliminate the adverse selection problem, leaving only a moral hazard problem.

We next consider the value of information when its conditional distributions fall somewhere between these two extremes and $a \in \{H, L\}$.

Lemma 2.4 (Value of Information about Ability) *Denote $A \equiv c\lambda(H - L)$ as in Lemma 1. Define $\bar{v} = \frac{1}{1+A}$ and $\bar{z}(\bar{v})$ implicitly as the solution to $\frac{g_H(\bar{z})}{g_L(\bar{z})} = \frac{1-v}{Av}$. For any joint cumulative information distribution $G(z)$ with conditional distributions $G_H(z)$ and $G_L(z)$ satisfying Assumption 1, the value of the information under a rational prior belief $v < \bar{v}$ is*

$$E[V(z|v < \bar{v})] = \left(2c\left(cR\sigma_y^2 + 1\right)\right)^{-1} \cdot \left[\begin{array}{c} \frac{A^2v^2}{1-v}\int_{-\infty}^{\bar{z}} \frac{[g_H(z) - g_L(z)]^2}{g_L(z)} dz + 2A\left(1 - \frac{Av}{1-v}\right)v\left(1 - G_H(\bar{z})\right) \\ -\left(1 - \left(\frac{Av}{1-v}\right)^2\right)(1-v)\left(1 - G_L(\bar{z})\right) \end{array}\right], \tag{2.20}$$

and the value of the information under a rational prior belief $v \geq \bar{v}$ is

$$E[V(z|v \geq \bar{v})] = \left(2c\left(cR\sigma_y^2 + 1\right)\right)^{-1} \cdot \left[\begin{array}{c} \frac{A^2v^2}{1-v}\int_{-\infty}^{\bar{z}} \frac{[g_H(z) - g_L(z)]^2}{g_L(z)} dz - 2A\left(1 - \frac{Av}{1-v}\right)vG_H(\bar{z}) \\ +\left(1 - \left(\frac{Av}{1-v}\right)^2\right)(1-v)G_L(\bar{z}) \end{array}\right]. \tag{2.21}$$

Proof. Please see the Appendix.

Note first that the threshold values $\bar{v}$ and $z(\bar{v})$ relate to the threshold belief from the solution to the principal's optimal contract problem shown in Section 3. The threshold $\bar{v}$ is the prior belief threshold. In Section 3, the threshold is expressed in terms of the general belief x, where x is either prior or posterior belief. Both thresholds take the same value. In the absence of information about type, the principal will offer the menu of contracts with a strictly positive PPS for type L and an information rent for type H when her prior belief falls below $\bar{v}$; otherwise, she will offer the menu with $\beta_L = 0$ and zero information rent. The threshold $z(\bar{v})$ is the minimum realized information value that, for a given prior v, causes the principal's posterior belief to exceed the belief threshold $\bar{v}$. Thus, any realization in excess of this value will result in her offering a menu of contracts with zero PPS for type L and zero information rent, but any realization below this value induces her to offer the menu with strictly positive values for these objects. The values of v and $z(v)$ vary in an intuitive way with various parameters such as the precision of the information.

The three additive terms in squared brackets respectively represent three mutually exclusive and collectively exhaustive possibilities with respect to the information. To see this more clearly, consider equation (2.20). The first term in the bracketed part of this equation, containing the integral, illustrates most directly the value of the information. It represents the expected gain from more accurately tuning the menu of contracts to the agent's probable type (which occurs when the realized value of the information falls below $\bar{z}$). It comprises the term $\frac{A^2v^2}{1-v}$, multiplied by a weighting factor that depends on the squared difference between the conditional information distributions for each possible realization of z. This term figures prominently in both equations (2.20) and (2.21): as a cost for information rent paid to the H type agent in the former and as a cost due to distortion in the effort induced of the L type agent in the latter. It is exactly the tradeoff between these two costs that the principal optimizes over when setting the menu of contracts. Thus, this first term appears in the expression for the value of the information because, with better information, the principal is on average better able to calibrate the menu of contracts toward the actual type of the agent she is dealing with.

The second and third terms arise because of the existence of the belief threshold. They represent the value of the information when its realized value exceeds $\bar{z}$. The second term, containing $2A$, is the expected increase in profit from eliminating the information rent for the type H agent when the realized value of the information exceeds $\bar{z}$, an event that occurs with probability $v(1 - G_H(\bar{z}))$. The third term is the expected loss in profit due to zero effort on the part of the L type agent (whose contract has zero PPS when the realized value is in this range), a loss that occurs with probability $(1 - v)(1 - G_L(\bar{z}))$. The corresponding terms in equation (2.21) have analogous interpretations.

The three terms in equations (2.20) and (2.21) are multiplied by a common term. The elements in this common term illustrate some of the important ways in which the adverse selection and moral hazard problems interact. The following propositions show how moral hazard and adverse selection affect the value of information.

Proposition 2.1 (Value of Information about Ability and the Severity of Moral Hazard) *The value of the information decreases in (1) the outcome noise (σ_y^2) and (2) the agent's riskaversion (R).*
Proof. The results follow directly from equation (2.20).

Proposition 2.2 (Value of Information about Ability and the Severity of Adverse Selection) *The value of the information is quasi-concave in (1) the post-contract relative outcome sensitivity to ability (λ) and (2) the dispersion of agent ability ($H - L$).*[18]
Proof. Please see the Appendix.

Proposition 2.1 is intuitive; outcome noise and risk aversion are two important factors that determine the severity of a moral hazard problem. The proposition, however, highlights the interaction between the moral hazard and adverse selection problems. The value of the information about ability is strictly decreasing in σ_y^2, the variance of the measure of the agent's effort given his ability, and in R, the agent's risk-aversion. Both of these objects appear in the problem only because the agent, who is risk-averse, is exposed

[18] This result assumes a constant quality of information—in particular, it abstracts from any effect that an increase in $H - L$ may have on the sensitivity of the information. See Section 4.3 for a discussion of how the sensitivity of information affects its value.

to risk due to the fact that his *effort* cannot be accurately measured—in fact, the agent's attitude toward risk does not bear directly on the adverse selection problem. However, because the principal has at her disposal only a single tool to motivate effort and to sort the agents according to ability, the principal faces an additional tradeoff beyond the tradeoff contained within the adverse selection problem *per se* that was discussed above. That is, in selecting a menu of contracts that more efficiently sorts the agents, the principal must lower the incentive to exert effort of at least one of the types (in this case, type L), which, all else equal, lowers her profits. For type H, when exposure to risk is particularly costly—whether because the agent is very risk-averse or because the production outcome is especially noisy—it becomes more costly for the principal to create variation in the PPSs offered. This makes sorting more difficult, which renders additional information about the agent's ability less valuable. By this indirect channel, the measurability of effort affects the value of information about the agent's type.

Because of the interaction between moral hazard and adverse selection, the interpretation of the role risk aversion plays is somewhat subtle. It should be noted that the result on risk aversion in Proposition 1 does not arise *solely* because an increase in R makes effort uniformly more costly to induce. If the principal were not attempting to sort agents, she would offer the same PPS to different types. Without sorting, risk aversion would not affect the value of information. To see this, consider the case in which both informed and uninformed principals disregard the type dispersion and offer the same PPS to all agents. Assume that the principal offers a contract that would be acceptable to both types.[19] The contract would be a combination of a salary and the same bonus scheme. The most profitable such contract would pay a salary and a bonus that would just meet the type L's reservation utility, which would give the H type an information rent that is proportional to $\lambda(H-L)$. Compare this outcome to that of a perfectly informed principal—this principal's expected profits would be higher by $v\lambda(H-L)$ (the information rent that would otherwise be paid to the H type times the probability that the agent is indeed H type) *irrespective of the importance of risk*, because neither principal here is using the PPSs to sort the agents. However, as in the standard case, the profits earned by both the uninformed and informed principals in this scenario would decline in the importance of risk due to the increased cost of inducing effort. This illustrates the fact that the sensitivity of the value of information to the importance of risk aversion does indeed arise from the dual function being served by the PPSs in the menu of contracts when the principal is simultaneously addressing both types of information problems.

Proposition 2.2 is less intuitive. It implies that the value of information initially increases in $\lambda(H-L)$ (the measure of the severity of adverse selection), but that this value peaks at some point and declines thereafter. It further implies that if adverse selection is extremely severe (because ability is very important in production and/or type differential $H-L$ is very large), the principal is not able to utilize the information as effectively as when the adverse selection problem is not as severe. The initial phase of increasing value occurs because when effort is important in production relative to ability, the principal benefits greatly from inducing near-optimal effort—in this case, from avoiding

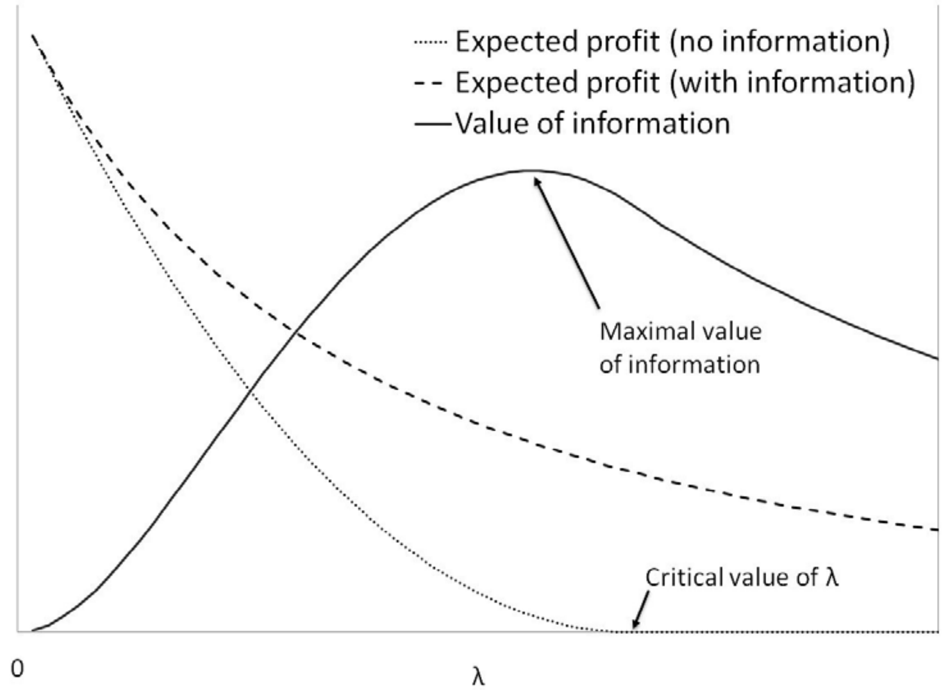
[19] If $1 < 2c\lambda(H-L)$, then it can be shown that the information rent that needs to be paid to the type H agent is higher than the marginal profit from his effort. In such a case, the principal does not try to induce any effort from either agent.

unnecessary distortion of the L-type contract. However, an increase in the severity of the adverse selection problem makes this more costly because a high PPS in the L-type contract becomes more attractive to the H type. Thus, while the uninformed principal's profits decline relatively steeply in $\lambda(H-L)$, the informed principal can "push back" by using the information she observes to offer a high PPS in the type L contract only when she is likely to be facing an L-type agent, so that her profits decline less steeply over this range, and thus the value of the information increases.

However, when ability is important in production relative to effort, it becomes relatively cheap for the principal to distort the L-type PPS in order to achieve the more valuable objective of sorting agents. Indeed, the uninformed principal will at some level of severity have fully distorted the menu of contracts, and after this point, her profits will remain constant in the severity of the adverse selection problem. But this is not so for the informed principal. Even when the adverse selection problem is very severe, she will sometimes observe information that causes her to (profitably) offer a menu of contracts that is not fully distorted. In other words, she will always have some chance to observe information that will cause her to behave differently from the uninformed principal. However, as $\lambda(H-L)$ increases, she will do so less and less frequently, which diminishes the expected value of the information.[20]

[20] Formally, the rate of decline of the uninformed principal's profits reaches zero at a finite value of $\lambda(H-L)$, while the rate of decline of the informed principal's profits approaches zero only in the limit. Thus, for some interval of high severity of adverse selection, the informed profits decrease more quickly in the degree of adverse selection than do the no-information profits, and the value of the information (which is simply the difference between the two) therefore declines in the degree of adverse selection over this interval as well.

Figure 2.1 Value of Information and Expected Profits as a Function of λ ∈ (0,8) H = 1, L = 0, v = 0.5, c = R = $\sigma_y^2 = \sigma_z^2 = 1$



The effect is illustrated in Figure 2.1. This figure plots the profits of the informed and uninformed principals, respectively, net of output due directly to agent ability (which output does not depend on the information) as a function of the relative marginal productivity of ability λ, fixing $H - L = 1$. The profits of the uninformed principal (dotted line) decline rather quickly with λ as she severely distorts the menu of contracts, while the informed principal (dashed line) is able to target her distortions away from the particular type of agent she is likely to be facing. The value of information, also shown in the Figure 1 (solid line[21]), is thus initially increasing in λ. However, for all values of λ greater than some critical value, the uninformed principal will offer an identical (maximally distorted) menu of contracts, and thus her expected profits will no longer

[21] The value of information line is on a different scale than the expected profit lines so that the reader can more easily observe the point of maximum value.

depend on λ in this range. Meanwhile, the informed principal will still sometimes observe information that causes her to profitably calibrate the contracts to some extent,[22] even for high values of λ, so that the expected profits earned by this principal still have some room to decline in the same range. But as λ increases, the informed principal observes such information with lower probability.[23] As a result, the difference between the two expected profits—that is, the value of information—diminishes over this range. An analogous graph plotting information value as a function of $(H-L)$ would be qualitatively identical.

2.4.2 The Importance of Information

Although the value of information about type is eventually decreasing in $\lambda(H-L)$, the importance of that information is in another sense uniformly increasing in the severity of the adverse selection problem. In particular, as the adverse selection problem increases in severity, the principal's expected profit from each agent type becomes more sensitive to the information. This can be seen by considering the *ex ante* versions of the expected profit function for each type with both y (the information about effort) and z (the as-yet unrealized information about agent type) as arguments. Using equations (2.14) and (2.15), we can write profits by agent type as functions of y and z as follows:

$$\begin{aligned}\pi_H &= \big(1-\beta_H(z)\big)y-\alpha_H(z)\\ &= \frac{cR\sigma_y^2}{cR\sigma_y^2+1}y - r_0 - \frac{A\left(1-A\frac{v}{1-v}LR(z)\right)}{c\left(cR\sigma_y^2+1\right)}\\ &\quad+\frac{1}{cR\sigma_y^2+1}\lambda H-\frac{\left(cR\sigma_y^2-1\right)}{2c\left(cR\sigma_y^2+1\right)^2},\end{aligned} \tag{2.22}$$

and

[22] In particular, she will still sometimes observe a value $z < \bar{z}$.

[23] In particular, z decreases in λ, making a realization $z < \bar{z}$ less likely.

$$\pi_L = (1-\beta_L(z))y - \alpha_L(z)$$
$$= \frac{cR\sigma_y^2 + A\frac{v}{1-v}LR(z)}{cR\sigma_y^2+1}y - r_0 - \frac{1-A\frac{v}{1-v}LR(z)}{cR\sigma_y^2+1}\lambda L$$
$$-\frac{\left(1-A\frac{v}{1-v}LR(z)\right)^2(cR\sigma_y^2-1)}{2c\left(cR\sigma_y^2+1\right)^2}, \qquad (2.23)$$

where $LR(z)$ is the likelihood ratio. (Note that these expressions hold only for realizations of z below the threshold, $\bar{z}$. Expected profits for realizations of z above this threshold do not depend on z, so they are not of interest here.) We focus on expected profits by evaluating the above equations at $y = E[y]$, setting output equal to its expected value (i.e., setting the realized output noise equal to its expected value: zero). This gives

$$E[\pi_H] = \lambda H - r_0 + \frac{1}{2c\left(cR\sigma_y^2+1\right)} - \frac{A\left(1-A\frac{v}{1-v}LR(z)\right)}{c\left(cR\sigma_y^2+1\right)}, \qquad (2.24)$$

and

$$E[\pi_L] = \lambda L - r_0 + \frac{1-\left(A\frac{v}{1-v}LR(z)\right)^2}{2c\left(cR\sigma_y^2+1\right)}. \qquad (2.24)$$

To see how the expected profits depend on the information about agent type, we take derivatives of these expressions with respect to $LR(z)$.[24] This gives

$$\frac{\partial E[\pi_H]}{\partial LR(z)} = \frac{A^2\frac{v}{1-v}}{c\left(cR\sigma_y^2+1\right)} > 0, \qquad (2.26)$$

And

[24] This is sufficient to establish the signs of these expressions by virtue of Assumption 1, and only the signs interest us here.

$$\frac{\partial E[\pi_L]}{\partial LR(z)} = -\frac{\left(A\frac{v}{1-v}\right)^2 LR(z)}{c\left(cR\sigma_y^2 + 1\right)} < 0. \tag{2.27}$$

Equation (2.26) shows that the expected profit when the agent is of type H increases in the realized z, as it should—a higher value of z more strongly suggests to the principal that the agent is type H, and her optimization given this information will increase profits when the agent is indeed type H. Likewise, equation (2.27) shows that the expected profit when the agent is of type L is decreasing in the realization of z, because a higher value of z causes the principal to hedge her bets toward the agent being H type, lowering her profits when she turns out to be wrong. Further analysis of these expressions leads to the following proposition:

Lemma 2.5 (Sensitivity of Expected Profit to Information about Ability) *The sensitivity of the expected profit from each agent type to information about the agent's type increases in the severity of the adverse selection problem.*[25]
Proof. Please see the Appendix.

The information about agent type (e.g., the likelihood that a particular agent is of type H) enables the principal to better calibrate the menu of contrasts, thereby increasing her expected profits. When the adverse selection problem becomes more severe (as $\lambda(H-L)$ increases), the information about the agent type becomes even more important in calibrating the menu. By designing a better tailored menu, the principal is able to increase her expected profits. In this sense, the importance of the information never diminishes. Lemma 2.5 underscores that the information about the agent's type becomes more important in determining the principal's expected profit as adverse selection becomes more severe. This result holds despite the fact that, as shown in Proposition 2.2, the value

[25] As noted above, the sensitivity is weakly increasing when $z > \bar{z}$.

of the information about agent type eventually decreases in the severity of the adverse selection problem.

Additionally, because the sensitivity of the value of information to $\lambda(H-L)$ is linked to the critical value beyond which the uninformed principal offers a maximally distorted menu of contracts, there is a relationship between the location of the peak value of information and the critical value:

Lemma 2.6 (Location of Peak Information Value) *Denote $A = c\lambda(H-L)$ as in Lemma 1. Define $\overline{\lambda(H-L)} \equiv \frac{1}{c}\left(\frac{1-v}{v}\right)$. Define further $[\lambda(H-L)]_{max}$ as the value of $\lambda(H-L)$ at which the information reaches its maximum value. Then the value of $\frac{[\lambda(H-L)]_{max}}{\overline{\lambda(H-L)}}$ depends only on the properties of the conditional information distributions and on no other parameters of the problem.*
Proof. Please see the Appendix.

Lemma 2.6 implies that, for a given distribution of information, the value of information peaks at a value of $\overline{\lambda(H-L)}$ that is a fixed "percentage" of the critical value of $\lambda(H-L)$. This in turn implies that any factor that increases this critical value (other than a change in the information distribution) will increase the range of values of $\lambda(H-L)$ over which the value of information is increasing:

Proposition 2.3 (Determinants of the Range of Increasing Value of Information) *For a fixed distribution of information, the range of values of $\lambda(H-L)$ over which the value of information increases is decreasing in c and v.*
Proof. Please see the Appendix.

That the range decreases in c (cost of effort) is intuitive. However, that it decreases in v is less so. It is because as v increases, the information about type becomes less important because the agent the principal is facing is more likely to be of type H and sorting becomes less valuable. Lemma 2.6 further implies that the value of information will be decreasing over some range above the critical value of $\lambda(H-L)$. Together with

Lemma 2.5, this means that there exists a range of $\lambda(H-L)$ over which the importance of the information in determining profit is increasing in the severity of adverse selection, while the value of the information is simultaneously decreasing in that severity. This occurs for the reason discussed earlier—that sorting by agent type, even though critical in determining expected profit, becomes very difficult when the adverse selection problem is particularly severe.

2.4.3 The Univariate Normal Conditional Information Distribution

We show in the previous section how the value of information depends on certain characteristics of the agents (the agents' risk-aversion, R, and the dispersion of agent ability, $H-L$); and on the characteristics of the production environment (the noisiness of output, σ_y^2, and the sensitivity of output to agent ability, λ). We now derive the value of information for a specific distribution of information: univariate normal conditional information distribution. This assumption allows us to perturb one moment of the distribution at a time by altering a single parameter while leaving the other fixed.[26] We expect similar qualitative results to hold for other distributions.

In particular, we assume that $g_a(z)$ is a normal distribution with mean θa and variance σ_z^2. Thus, the information distribution conditional on the agent being of ability a is

$$g_a(z) = \frac{1}{\sqrt{2\pi\sigma_z^2}} exp\left[-\frac{(z-\theta a)^2}{2\sigma_z^2}\right] \quad \text{for} \quad a \in \{H, L\}. \tag{2.28}$$

[26] Kim and Suh (1991) show that, in a moral hazard setting, information with a conditional normal distribution can be unambiguously ranked in quality.

Note that the greater is θ, the larger is the difference in the means of the information distributions generated by the two agent types. Thus, θ can be interpreted as the sensitivity of the information to the agent's ability. The inverse of the distributions' variance $1/\sigma_z^2$ can be interpreted as the precision of the information. Note too that the conditional distributions differ in their means but not in their variances.

Lemma 2.7 (The Value of Univariate Normally Distributed Information) *Suppose the conditional information distributions are normal as specified in equation (2.28). Define A ≡ cλ(H − L) as in Lemma 1. Define* $\bar{v}$ *as in Lemma 2.4. Define* $\overline{z}(v) = \frac{\theta(H+L)}{2} + \frac{\sigma_z^2}{\theta(H-L)} log\left(\frac{1-v}{Av}\right)$*, also as in Lemma 2.1. Let Φa(·) be the cdf of the normal distribution with mean θa and variance* σ_z^2*. The value of the information under a rational prior belief* $v < \bar{v}$ *is*

$$E[V(z|\, v \le \overline{v})] = \left(2c\left(cR\sigma_y^2 + A\right)\right)^{-1}$$

$$\cdot \begin{bmatrix} A^2 \dfrac{v^2}{1-v}\left(e^{\frac{\theta^2(H-L)^2}{\sigma_z^2}} \Phi_{2H-L}(\overline{z}) - \left(2\Phi_H(\overline{z}) - \Phi_L(\overline{z})\right)\right) \\ +2A\left(1 - A\dfrac{v}{1-v}\right) v\left(1-\Phi_H(\overline{z})\right) - \left(1 - \left(A\dfrac{v}{1-v}\right)^2 (1-v)\left(1-\Phi_L(\overline{z})\right)\right) \end{bmatrix}, \quad (2.29)$$

and the value of the information under a rational prior belief $v \ge \bar{v}$ *is*

$$E[V(z|\, v \ge \overline{v})] = \left(2c\left(cR\sigma_y^2 + A\right)\right)^{-1}$$

$$\cdot \begin{bmatrix} A^2 \dfrac{v^2}{1-v}\left(\begin{array}{c} e^{\frac{\theta^2(H-L)^2}{\sigma_z^2}} \Phi_{2H-L}(\overline{z}) \\ -\left(2\Phi_H(\overline{z}) - \Phi_L(\overline{z})\right)\left[-2A\left(1 - A\dfrac{v}{1-v}\right) v\Phi_H(\overline{z})\right. \end{array}\right) \\ +\left(1 - \left(A\dfrac{v}{1-v}\right)^2 (1-v)\Phi_L(\overline{z})\right) \end{bmatrix}, \quad (2.30)$$

Proof. The result follows directly from application of Lemma 2.4 to the specified distribution.

This result leads immediately to the following proposition.

Proposition 2.4 *The value of the information increases in (1) its precision* $(1/\sigma_z^2)$*; and (2) its sensitivity (θ).*
Proof. Please see the Appendix.

We first discuss the result in Proposition 2.4 regarding the information's precision. As the precision of the information increases (that is, as σ_z^2 decreases), the threshold $\bar{z}$ approaches the average agent ability. This occurs because as the information becomes nearly perfectly precise, realizations of the information suggesting that agent ability is on one or the other side of the average becomes increasingly strong evidence that the agent's true ability also lies on that side of the average. As the precision increases without bound, the value of the information approaches its maximum, which corresponds to the problem collapsing into one of strictly moral hazard.

Further, note that when $v < \bar{v}$, the value of z increases and when $v \geq \bar{v}$, decreases in the variance of the information. This reflects the fact that, as the information becomes less informative, it has less effect on the principal's prior. Thus, when $v < \bar{v}$, $\lim_{\sigma_z^2 \to \infty} \Phi(\bar{z}) = 1$, and when $v \geq \bar{v}$, $\lim_{\sigma_z^2 \to \infty} \Phi(\bar{z}) = 0$ for any normal density function. Examining equations (2.29) and (2.30), we can see immediately that these two facts imply that the value of the information approaches zero as its variance increases without bound, as expected.

Regarding the effect of sensitivity of the information to ability on the value of the information, note that the threshold $\bar{z}$ is non-monotonic in θ; it increases without bound as θ also increases without bound *and* as θ approaches zero. However, the former effect arises only because an increase in the sensitivity as we have defined it here also raises the average of the means of the conditional information distributions (which corresponds to average agent ability in the discussion of precision above). After taking account of this fact, the effect of increasing sensitivity on the threshold mirrors that of increasing precision—the threshold approaches the (modified) average agent ability because a realization of the information on one or the other side of the average becomes increasingly convincing evidence of the agent's true ability. As θ increases without bound, the value of the information again approaches its maximum.

2.5 Conclusion

The analytical accounting research literature has traditionally emphasized moral hazard in the context of executive compensation and performance measurement, yet uncertainty regarding managerial ability may also play an important role at the top level of an organizational hierarchy. In many situations, executive abilities to make swift, decisive, and strategic decisions are more important than putting in long hours. McKinsey & Company (2018) emphasizes the importance of talent management by matching the right talent to the right role as a crucial ingredient of corporate success. Their survey found that "organizations with effective talent-management programs have a better chance than other companies of outperforming competitors." Given that managers are likely to possess different levels of ability, sorting managers for their ability is an important concern for principals. In a comprehensive survey of theory and evidence on executive compensation, Edmans, Gabaix, and Jenter (2017) call for more research on "the theories that combine both learning about ability and moral hazard, or empirical studies that analyze the relative importance of learning versus moral hazard for observed contracts, would be valuable." In this paper, by studying a setting with both moral hazard and adverse selection to derive the value and optimal use of information about ability in the design of compensation contracts, we contribute to the area of research called for by Edmans et al.

We derive the value to the principal of information about an agent's ability and identify how various features of the environment interact in determining its value. Our results highlight how moral hazard and adverse selection interact with each other. The value of information about agents depends on certain characteristics of the agents (e.g., risk and effort aversion, and ability dispersion) and of the information and production environment. Information that improves the principle's knowledge of the agent's type is valuable because such knowledge enables the principal to more accurately calibrate the menu of contracts to offer to the candidate. We prove that the value of the information decreases in the noisiness of the outcome measure and the risk aversion of the agent, due to the fact that the principal has only PPS at her disposal to simultaneously sort agents and induce their effort, resulting in a tradeoff between these two goals. The principal trades off the information rent to be paid to the type H agent against the reduction in (effort) incentives for the type L agent. This tradeoff becomes more costly as the severity of adverse selection increases. As a result, we find that the value of the information initially increases, peaks, then decreases in measures of the severity of adverse selection. This is due to the fundamental limit that the principal faces in distorting the contracts she offers. However, despite its decreasing value, the information plays a bigger role in determining the principal's expected profits when adverse selection is more severe. This result implies that there will be situations where the value of the information decreases in the severity of adverse selection, while its importance in determining expected profits increases in that severity. We also prove that the value increases in both the sensitivity and precision of the information.

These results should be contrasted to the value of information in a moral hazard setting. In such a situation, a principal would be interested in information or signals that are informative of the effort exerted by the agent. More accurate information that can be used as a performance measure enables the principal to motivate the agent with a lower level of risk imposed on the agent. As a result, risk-sharing between the principal and the agent improves. At the extreme where a performance measure perfectly identifies effort expended, the principal can offer a contract with a salary that is paid if and only if the agent exerts the pre-specified first-best effort level. The salary covers the agent's reservation utility and the cost of effort. Since the agent is not exposed to any risk, the principal does not have to pay any risk premium. As this example shows, the value of information in moral hazard settings monotonically increases in the accuracy of information.

Our results for the adverse selection setting underscore the value of pre-contract information. Generally, principals will be more willing to incur search costs that inform them about candidates' abilities when the value of information is sufficiently high. An interesting implication of our result that the value of information is quasi-concave in the severity of adverse selection is that the principal might want to engage in search activities that can reduce the type dispersion if the severity is beyond the peak point. While we focused on the acquisition of information that assesses the agent type more accurately, under some conditions the principal will prefer a pool of applicants in which the distribution of the types is not too wide. In many recruiting situations, applicants are pre-sorted based on some clearly and readily observable traits (e.g., the education level and the quality of schools). Such sorting can be viewed as an effort to narrow the type dispersion to a reasonable level (i.e., within the range in which the value of information is increasing) before the principal gathers more information about individual candidates. Our analysis also has an implication regarding whether to search for a new executive internally or externally. Inside candidates offer richer precontract information, while outside candidates might come from a pool of higher-average talents. Engaging a head-hunting firm also changes the pool of available candidates, at a cost.

Finally, although we speak throughout the paper in terms of a standard principal-agent relationship, and we equate *type* with *ability*, our results can be readily adapted to other settings that entail both adverse selection and moral hazard. For example, instead of a principal-owner negotiating a salary and a contingent bonus with a prospective agent-manager who has private knowledge of his own ability, consider a principal-investor negotiating with an agent-entrepreneur/manager over the terms of acquiring his startup company, of which only the agent knows the true quality. Our model can accommodate this scenario by reinterpreting the salary as a fixed component of the purchase agreement and the bonus as a contingent component of the agreement, where the agent commits to manage the company for a specified period of time after the acquisition, and where the contingent payment will depend on the firm's performance during this post-acquisition period. Our model has interesting empirical predictions in this case as well, including the type of information the principal will be willing to bear a cost to obtain (or the agent will be willing to bear a cost to signal), how the terms of such an agreement should depend on the features of the firm and the information environment, and in which types of industries (and for which types of firms) such agreements should be more relevant and likely to be adopted.

CHAPTER 3 THE RELATIVE WEIGHTS OF ABILITY SIGNALS IN THE PRESENCE OF MORAL HAZARD AND ADVERSE SELECTION

3.1 Introduction

Incentive contracts serve two purposes: to resolve moral hazard and adverse selection problems arising from unobservable managerial effort and ability, respectively. Most of the analytical accounting literature on executive compensation and performance measurement has focused on problems involving moral hazard (Banker and Datar 1989; Bushman and Indjejikian 1993; Feltham and Xie 1994; Datar et al. 2001). While a few studies consider adverse selection problems (Biglaiser and Mezzetti 1993; Darrough and Melumad 1995; Antle and Fellingham 1995; Lazear, 2000; Inderst 2001; Dutta 2008), none examines the relative weights placed on performance measures in the presence of adverse selection. Moreover, because managerial ability is difficult to discern, firms may utilize ability signals in the design of their compensation schemes. Such signals include measures of human capital (e.g., education and experience), as well as information gleaned from informal social interactions along with the formal interview and selection process. Nevertheless, the agency literature has not ascertained how ability signals should be incorporated in the design of incentive contracts; nor has the literature ascertained whether the principal should rely more on ability signals versus traditional performance measures.

There are at least two reasons why managerial ability is important to performance measurement. First, pay-performance sensitivity is independent of managerial ability in agency models without adverse selection (e.g., Banker and Datar 1989; Feltham and Xie 1994). Yet, a large stream of empirical studies document that managerial ability is a major determinant of executive compensation (e.g., Hambrick and Mason 1984; Tushman and Romanelli 1985; Agrawal and Knoeber 2001; Adams et al. 2005; Gabaix and Landier 2008; Tervio 2008; Malmendier and Tate 2009).

We model the agency relationship between the owner of a firm (the principal) and the manager of the firm (the agent) when the manager exerts unobservable effort to operate the firm (moral hazard) and the manager is endowed with unobservable expertise that is valuable to the firm (adverse selection). In this paper, we develop a model to analyze the value of pre-contract information in a principal-agent setting with both moral hazard and adverse selection. A moral hazard problem may emerge if a principal cannot observe her agent's action, such as his effort level. The principal can address this problem by making the manager's compensation a function of some observable outcome that depends on the manager's effort (typically referred to as pay-performance sensitivity or PPS). An adverse selection problem may arise when some relevant traits of the agent cannot be observed by the principal (e.g., the manager's ability) so that the agent can misrepresent himself. The principal can address this type of problem by offering a menu of contracts, each of which would only be chosen by an agent of a particular level of ability.

We address the issue of how to optimally use various pieces of information about an agent's ability. Given that a principal usually has several pieces of information, such as the agent's education and past performance, the principal needs to aggregate the pieces by assigning relative weights. We show that, not surprisingly, the weights depend on the quality (precision and sensitivity) of the information. Interestingly, the analysis points out that the weights depend on the distribution characteristics of the information and not on firm- or agency-specific attributes, suggesting that relative weights used by different firms can be identical. This finding may be useful, especially for empirical work.

Our study builds on a large body of earlier work on how to resolve conflicts in the principal agent relationship, which arise due to the separation of ownership and management (Jensen and Meckling 1976; Fama 1980). Agency theory addresses the problems of information asymmetries (e.g., unobservable effort and agent type) between principals (owners) and agents (managers) by designing incentive contracts that help align their interests (Jensen and Zimmerman 1985; Eisenhardt 1989). In moral hazard settings, performance-based compensation enhances goal congruence, motivating agents to work hard so as to increase the payoff of the principal (e.g., Holmström 1979; Banker and Datar 1989; Bushman and Indjejikian 1993). Contemporaneous performance measures are useful because they provide information about the agent's unobservable effort (e.g., Holmström 1979; Banker and Datar 1989; Feltham and Xie 1994). For example, Holmström (1979) proposes that the principal can increase her expected payoff by explicitly including various performance measures in the contract, if the sufficient statistic condition is violated.

In the accounting literature, a number of analytical papers examine the relative weights that should be placed on multiple performance measures in a moral hazard context. Banker and Datar (1989) study how to optimally combine multiple *ex post* signals regarding agent effort, and show how the relative importance of signals depends on its statistical properties. Feltham and Xie (1994), Datar, Kulp, and Lambert (2001), and Christensen, Şabac, and Tian (2010) study a setting with multiple actions and performance measures. The focus of these analyses is on the allocation of effort across actions and, in assigning weights to the signals, on the tradeoff between (a) the congruity between the agent's overall compensation and the outcome of interest to the principal, and (b) the precision of the signal. Amershi, Banker, and Datar (1990) model the principal-agent relationship with multiple signals and examine the payoff to the principal in terms of statistical and economic sufficiency of aggregations of the signals. They argue that the class of distributions cited in Holmström's result (i.e., that aggregates must be sufficient statistics for the individual signals) is a subclass of a broader class of distributions. They also show that outside that subclass, it is possible to aggregate accounting signals without any economic loss to the principal. Kim and Suh (1991) propose that the variance of the likelihood ratio can rank any information system in the traditional moral hazard framework. They also link their results to the sensitivity and precision of a signal when a separability condition is satisfied. Empirical research in accounting has shown that both accounting and market measures of contemporaneous performance receive positive weights in compensation contracts and the weights are determined by sensitivity and noise of the performance measures (e.g., Lambert and Larcker 1987; Sloan 1993; Evans, Kim, and Nagarajan 2006). Furthermore, there is extensive evidence that executives are rewarded on the basis of various financial performance measures (e.g., Healy 1985; Bushman, Indjejikian, and Smith 1996; Ittner, Larcker, and Rajan 1997; Core, Guay, and Verrecchia 2003).

We propose that empirical studies of executive compensation should clearly distinguish between traditional performance measures, such as market and accounting returns, versus ability measures. It is important to distinguish between performance versus ability measures since, as proved in this paper, they serve different functions in the optimal compensation scheme when we recognize the fact that firms face both moral hazard and adverse selection problems. Our analysis should provide guidance as to how to evaluate the value of information about ability, which will be useful for the principal in deciding how much to invest in acquiring such information and how to use the acquired information. In addition, a principal possessing multiple pieces of information would want to aggregate the information, and our analysis can be extended to handle this case. While we have treated z as a scalar for simplicity in Chapter 2, we relax this assumption in this chapter and address the question of how to assign relative weighs to different pieces of information, such as the agent's educational background, professional qualifications, and past performance in the firm he worked, or in the current firm.

The remainder of the paper is organized as follows. Section 3.2 describes the model. Section 3.3 provides the optimal solution to the model. Section 3.4 presents the main results and discusses the empirical ramifications. Section 3.5 concludes.

3.2 The Agency Model

A risk-neutral principal (owner) wishes to contract with a risk- and effort-averse agent (manager) to operate her firm. After a contract is agreed upon, the agent exerts unobservable effort e, which represents any action undertaken by the agent on behalf of the principal. The agent is endowed with ability $a \in \{H,L\}$, with $H > L$, the true value of which is known only to the agent.[27]

[27] We refer to agents of the two different levels of ability as "types." While agents know their type, we assume in our paper that they are not able to credibly signal their type, or equivalently that any such signaling is already incorporated in the principal's prior beliefs.

Thus, the principal faces a moral hazard problem (with respect to effort e) and an adverse selection problem (with respect to managerial ability a). The principal's rational (that is, correct) prior belief (i.e., pre-contract information) about the agent's ability is $P(a = H) = v$ and $P(a = L) = 1 - v$.

We adopt the standard framework that supposes a linear contract, exponential utility on the part of the agent, and normal disturbance to firm output (see Holmström and Milgrom 1987; Bose, Pal, and Sappington 2011). Our main results regarding the value of information, however, do not depend on these assumptions. An agent endowed with ability a who exerts effort e generates the outcome:[28]

$$\tilde{y}(e,a) = \gamma_a a + \gamma_e(e + \epsilon).$$

Dividing the equation by γ_e yields

$$y(e,a) = \frac{\tilde{y}(e,a)}{\gamma_e} = \frac{\gamma_a}{\gamma_e} a + e + \epsilon$$

$$= \lambda a + e + \epsilon, \tag{3.1}$$

where $\frac{\gamma_a}{\gamma_e}$ is relabeled as λ and ϵ is a mean-zero normal disturbance term with variance σ_ϵ^2. $\lambda \geq 0$ represents the productivity of ability (or the sensitivity of the outcome to the agent's ability), and the productivity of effort (the sensitivity of the outcome to the agent's effort) is normalized as one. Because ability is not directly observable by the principal, the parameter λ also relates to the importance of the agent's private information in the principal-agent relationship (that is, to the importance of the adverse selection

[28] We adopt this formulation, where the disturbance is multiplied by the marginal productivity of effort, so that in our modified production function, we can change the relative productivity parameter without affecting the noisiness of the output. In the alternate specification, changes in the marginal productivity of effort affect the value of information in part by changing the informativeness of the realized output regarding effort, something that is not of direct interest.

problem). Note that the outcome function is separable in ability and effort. This implies, for example, that the productivity of effort is independent of ability or agent's type.[29]

The principal designs a menu of compensation contracts contingent on the outcome y that induces the agent, irrespective of his ability, to exert the desired level of effort (incentive compatibility), truthfully reveal his ability (truth-telling), and voluntarily sign the contract (individual rationality). The principal adopts compensation contracts of the form:

$$w_a(y) = \alpha_a + \beta_a y \text{ for } a \in \{H, L\}, \tag{3.2}$$

where α_a represents the salary intended for an agent of type a and β_a represents the PPS for the same agent type. The objective of the principal is to maximize the expected outcome net of the agent's compensation. Although we suppress them here, the values of y, α_a, and β_a, in general, will also be functions of other parameters of the problem.

The principal may be able to observe some information about the agent's ability prior to negotiating the contract. The conditional distribution of the information $\mathbf{z} \in \mathbf{Z}$ given the agent's true ability a is denoted $g(\mathbf{z}|a)$ for each $a \in \{H,L\}$, where $\mathbf{z}$ is a vector of n pieces of component information $(z_1,z_2,\cdots,z_n)$. We also use the notation $g_a(\mathbf{z}) \equiv g(\mathbf{z}\,|a)$ for the remainder of the paper, and similarly $G_a(\mathbf{z}) \equiv G(\mathbf{z}\,|a)$ for the corresponding cumulative distribution function. If she observes information, the principal updates her

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[29] The separability assumption is not as restrictive as it may appear. Suppose that the outcome is Cobb-Douglas, $y(e,a,z) = a^{\gamma_a}(e+\varepsilon)^{\gamma_e}$. Taking logs, this becomes $\ln y(e,a,z) = \gamma_a \ln a + \gamma_e \ln(e+\varepsilon)$. Hence, if we redefine the outcome, effort, and ability in log terms, then we obtain a linear production function. The only difference is in the interpretation of the parameters: in the Cobb-Douglas case, γ_e represents the elasticity of the outcome with respect to effort, while in the linear case, it represents the sensitivity of the outcome to effort.

belief about the distribution of the agent's ability by using Bayes' rule.[30] Thus, for a realized value of z, the density function $f(a|\mathbf{z})$ representing the principal's posterior belief is in general given by

$$f(a|\mathbf{z}) = \frac{g_a(\mathbf{z})f(a)}{\int_\Omega g_a(\mathbf{z})f(a)da}.$$

In our discrete ability setting, $\Omega = \{H,L\}$, this becomes

$$P(a|\mathbf{z}) = \frac{g_a(\mathbf{z})P(a)}{vg_H(\mathbf{z}) + (1-v)g_L(\mathbf{z})}. \qquad (3.3)$$

where $P(H) = v>0$ and $P(L) = 1 - v>0$ are the principal's prior beliefs. This implies that

$$\frac{u}{1-u} = \frac{v}{1-v}\frac{g_H(\mathbf{z})}{g_L(\mathbf{z})} = \frac{v}{1-v}LR(\mathbf{z}), \qquad (3.4)$$

where $u(\mathbf{z}) \equiv P(a{=}H|\,\mathbf{z})$ and $1-u(\mathbf{z}) = P(a{=}L|\,\mathbf{z})$ represent the principal's posterior beliefs, and $LR(z)$ is the likelihood ratio.[31]

The probability density function $g_a(\mathbf{z})$ represents the frequency of observing information $\mathbf{z}$ when the agent is type a. Thus, the ratio $\frac{g_H(\mathbf{z})}{g_L(\mathbf{z})} = LR(\mathbf{z})$ is the relative likelihood of observing information z when the agent is of type H compared to when the agent is of type L. The principal updates her prior on this ratio according to Bayes' rule by multiplying it by the relative likelihood as shown in equation (3.4). We make the standard assumption below regarding the likelihood ratio (Milgrom 1981), which we maintain throughout:

[30] Milbourn (2003) also studies a setting in which the principal updates her belief about agent's ability upon observing information.

[31] For the rest of the paper, we suppress the argument $\mathbf{z}$ in the function u($\mathbf{z}$).

Assumption 3.1 (Monotone Likelihood Ratio Property for Information) *The likelihood ratio LR(**z**) is increasing in **z**.*

The agent's preferences exhibit constant absolute risk aversion (CARA) with coefficient R (see Dutta 2008, Datar, Kulp, and Lambert 2001, Holmström and Milgrom 1987, and Feltham and Xie 1994). The agent exerting effort e incurs a dollar-equivalent cost $C(e) = ce^2/2$.[32] As such, the certainty equivalent value of the contract specifying the pair (α,β) designed for an agent of type a consists of a salary and bonus minus the costs of risk and effort:

$$CE(\alpha,\beta) = \alpha_a + \beta_a(\lambda a + e) - \frac{R\beta_a^2\sigma_y^2}{2} - \frac{ce^2}{2}, a \in \{H, L\}. \tag{3.5}$$

We denote the agent's reservation utility as r_0.[33] We discuss some implications of relaxing the assumption that both types have the same reservation utility below. We maintain the assumption that $r_0 < \lambda L$, which ensures that the principal will find it profitable to employ the type L agent even when he exerts zero effort. Relaxing this assumption complicates the analytical results without changing them qualitatively.[34]

Assumption 3.2 (Natural Exponential Family for Information Distribution) *The probability density function for information **z** conditional on ability a is of the form*

$$g_a(z) = exp\left[\sum_{i=1}^{n} p_i\, az_i - \sum_{i=1}^{n} d_i(a) + s(z_1, \cdots, z_n)\right]. \tag{3.6}$$

[32] As in Dutta (2008) and Feltham and Xie (1994), the quadratic cost function assumption is made for convenience. All results continue to hold for a general cost function that is increasing and convex.

[33] Dutta (2008) relaxes the assumption that agent type and reservation utility are independent.

[34] In particular, relaxing this assumption leads to a range of beliefs over which the principal will exclude the L type agent and will offer the H type agent the pure moral hazard contract. The principal's profits in this scenario closely mimic those in the case we study here.

Note that under Assumption 3.2, $p_i > 0$ for all $i = 1,...,n$ is necessary and sufficient to satisfy Assumption 1. Among the most commonly used distributions that satisfy Assumptions 1 and 2 are the Normal, Gamma, Poisson, Binomial, and Chi-squared distributions.

The timing of events in this model is as follows. First, the principal may observe information about the agent's ability, in which case she updates her beliefs accordingly. She then designs the compensation mechanism, which consists of a menu of contracts containing two contracts each of which is intended for a particular agent type.[35] After observing the menu, the agent decides what ability to report to the principal (which in equilibrium is his actual ability) and whether to accept the corresponding employment contract (which, in equilibrium, he does). The contract is signed, and the agent decides what level of effort to exert. Finally, the outcome is realized and is divided between the principle and the agent according to the terms of the contract.

3.3 The Principal's Mechanism Design Problem under Moral Hazard and Adverse Selection

The principal's problem is to maximize the expected outcome net of the agent's compensation, given her beliefs regarding his ability. Suppose that the principal believes the agent is of type H with probability x. For an uninformed principal, $x = v$, while for an

[35] By appealing to the revelation principle, we solve for a truth-inducing menu of contract. Compensation mechanisms in practice may not be a direct mechanism and may involve non-truthful reporting. There are alternative optimal mechanisms characterized by complex communication between the principal and agent, and in which the agent lies in equilibrium about his ability. See Ronen and Yaari (2001) who develop a model with an explicit nontruth-telling equilibrium. Indeed, it may be more realistic in some cases to imagine the process of interview and negotiation between an owner and a prospective manager to unfold in this way.

informed principal, who acquires additional information about the agent, $x = u$, where v and u are prior and posterior beliefs about the type. Then the principal solves

$$\max_{\{\alpha_a,\beta_a,e(a)\}} \begin{array}{c} xE\left[y(e(H),H) - w\left(H, y(e(H),H)\right)\right] \\ +(1-x)E\left[y(e(L),L) - w\left(L, y(e(L),L)\right)\right] \end{array}, \tag{3.7}$$

subject to

$$e(a) = \arg\max_{\tilde{e}} CE(\alpha_a, \beta_a | a) \text{ for all } a \in \{H, L\}, \tag{3.8}$$

$$CE(\alpha_a, \beta_a | a) \geq r_0 \text{ for all } a \in \{H, L\}, \tag{3.9}$$

$$CE(\alpha_a, \beta_a | a) \geq CE(\alpha_{\tilde{a}}, \beta_{\tilde{a}} | a) \text{ for all } a, \tilde{a} \in \{H, L\}, \tag{3.10}$$

where $CE(\alpha_{\tilde{a}}, \beta_{\tilde{a}} | a)$ is the certainty equivalent of an agent of type a when claiming to be of type $\tilde{a}$ and optimizing accordingly. Equation (3.8) is the agent's incentive compatibility (IC) constraint, equation (3.9) is the individual rationality (IR) constraint, and equation (3.10) is his truth-telling (TT) constraint. We also impose $e(a) \geq 0$ for $a \in \{H,L\}$. Note that the principal is willing to hire either a type H or type L agent. Her mechanism design problem is to derive a menu of contracts designed judiciously for each type.

We first provide the solution to the principal's mechanism design problem under both moral hazard and adverse selection in the following lemma. We then discuss two special cases: pure moral hazard and pure adverse selection.

Lemma 3.1 (Solution to the Principal's Problem) *Suppose the principal rationally believes that the agent is of type H with probability x, where $x \in \{v,u\}$ and $0<x<1$. Denote $A \equiv c\lambda(H - L)$. Further, define $\bar{x} \equiv \frac{1}{A+1}$. Then the optimal pay-performance sensitivity (PPS) for the H type agent is*

$$\beta_H = \frac{1}{cR\sigma_y^2 + 1}, \tag{3.11}$$

and the optimal PPS for the L type agent is

$$\beta_L = \begin{cases} \dfrac{1}{cR\sigma_y^2 + 1} & if\, 0 < x < \bar{x} \\ 0 & if\ \bar{x} \leq x < 1 \end{cases}, \tag{3.12}$$

The expected total compensation of the H type agent is

$$E[w_H] = \begin{cases} r_0 + \dfrac{1}{2c(cR\sigma_y^2 + 1)} + \dfrac{2A\left(1 - A\frac{x}{1-x}\right)}{2c(cR\sigma_y^2 + 1)} & if\, 0 < x < \bar{x} \\ r_0 + \dfrac{1}{2c(cR\sigma_y^2 + 1)} & if\ \bar{x} \leq x < 1 \end{cases}, \tag{3.13}$$

and the expected total compensation of the L type agent is

$$E[w_L] = \begin{cases} r_0 + \dfrac{\left(1 - A\frac{x}{1-x}\right)^2}{2c(cR\sigma_y^2 + 1)} & if\, 0 < x < \bar{x} \\ r_0 & if\ \bar{x} \leq x < 1 \end{cases}, \tag{3.14}$$

The expected profit of the principal when she is matched with an H type agent is

$$E[\pi_H] = \begin{cases} \lambda H - r_0 + \dfrac{1}{2c(cR\sigma_y^2 + 1)} - \dfrac{2A\left(1 - A\frac{x}{1-x}\right)}{2c(cR\sigma_y^2 + 1)} & if\, 0 < x < \bar{x} \\ \lambda H - r_0 + \dfrac{1}{2c(cR\sigma_y^2 + 1)} & if\ \bar{x} \leq x < 1 \end{cases}, \tag{3.15}$$

and the expected profit of the principal when she is matched with the L type agent is

$$E[\pi_L] = \begin{cases} \lambda L - r_0 + \dfrac{\left(1 - A\frac{x}{1-x}\right)^2}{2c(cR\sigma_y^2 + 1)} & if\, 0 < x < \bar{x} \\ \lambda L - r_0 & if\ \bar{x} \leq x < 1 \end{cases}, \tag{3.16}$$

The expected profit of the principal over ability is then

$$E_a[\pi] = \begin{cases} \lambda(xH + (1-x)L) - r_0 + \dfrac{x}{2c(cR\sigma_y^2 + 1)} \\ -x\dfrac{2A\left(1 - A\frac{x}{1-x}\right)}{2c(cR\sigma_y^2 + 1)} + (1-x)\dfrac{\left(1 - A\frac{x}{1-x}\right)^2}{2c(cR\sigma_y^2 + 1)} & if\, 0 < x < \bar{x} \\ \lambda(xH + (1-x)L) - r_0 + \dfrac{x}{2c(cR\sigma_y^2 + 1)} & if\, \bar{x} \leq x < 1 \end{cases}, \tag{3.17}$$

Proof. Please see the Appendix.

3.4 Relative Weights and Aggregation of Information

The following lemma proves that under this broad class of distributions, the principal can linearly aggregate information in contracting without loss of generality.

Proposition 3.1 (Aggregation of Information) *Suppose the density of the conditional information distribution meets Assumptions 3.1 and 3.2. Then the optimal compensation*

contract combines the information using the linear aggregation $\sum_{i=1}^{n} p_i z_i$, *where* p_i, $i = 1,\ldots,n$, *are the optimal weights.*
Proof. Please see the Appendix.

Proposition 3.1 can be interpreted in terms of statistical sufficiency. The general form of the conditional density function of distributions satisfying Assumption 3.2 can be expressed as

$$g_a(\boldsymbol{z}) = \exp[T(\boldsymbol{z})a]\, h(\boldsymbol{z}),$$

Where $T(\boldsymbol{z}) = \sum_{i=1}^{n} p_i z_i$ is a sufficient statistic for a.[36] This result has strong practical empirical implications. It suggests that not only is it possible to summarize many individual observations of a given type of information, but it is also possible to aggregate various types of information that are likely to have quite different distributions (such as education, performance of previously managed firm, or number of instances of positive media coverage) without loss of generality, as long as each of the distributions of those various pieces of information belongs to the family of commonly encountered distributions satisfying Assumption 3.2.

Proposition 3.2 (Optimal Relative Weights in Adverse Selection, Uncorrelated) *Suppose the principal observes two uncorrelated pieces of information* z_1 *and* z_2, *with a joint distribution meeting Assumption 3.2. Define the precisions of* z_1 *as* $\rho_1^2 \equiv \frac{1}{Var(z_1)}$ *and* z_2 *as* $\rho_2^2 \equiv \frac{1}{Var(z_2)}$. *Then the optimal relative impacts of the information* z_1 *and* z_2 *on the expected compensation for an agent of type* $a \in \{H,L\}$ *is proportional to: the precision of the information and sensitivity of the information for uncorrelated information, i.e.,*

$$\frac{\partial w(a; z_1, z_2)/\partial z_1}{\partial w(a; z_1, z_2)/\partial z_2} = \frac{\mu_1}{\mu_2}\frac{\rho_1^2}{\rho_2^2}; \tag{3.18}$$

Proof. Please see the Appendix.

[36] Note too that our aggregation result holds not just for identically distributed information, but for a large class of correlated and non-identically distributed information as well.

Proposition 3.3 (Optimal Relative Weights in Adverse Selection, Correlated) *Suppose the principal observes two pieces of information* z_1 *and* z_2*, with a joint distribution meeting Assumption 3.2. Define the precisions of* z_1 *as* $\rho_1^2 \equiv \frac{1}{Var(z_1)}$ *and* z_2 *as* $\rho_2^2 \equiv \frac{1}{Var(z_2)}$*; define their (unadjusted) sensitivities as* $\mu_1 \equiv \partial E(z_1)/\partial a$ *and* $\mu_2 \equiv \partial E(z_2)/\partial a$*; and define their adjusted sensitivities as* $\zeta_1 \equiv \mu_1 - \kappa\rho_1^2\mu_2$ *and* $\zeta_2 \equiv \mu_2 - \kappa\rho_2^2\mu_1$*, respectively, where* $\kappa \equiv cov(z_1, z_2)$*. Then the optimal relative impacts of the information* z_1 *and* z_2 *on the expected compensation for an agent of type a ∈ {H,L} is proportional to: the precision of the information and adjusted sensitivity of the information for correlated information, i.e.,*

$$\frac{\partial w(a; z_1, z_2)/\partial z_1}{\partial w(a; z_1, z_2)/\partial z_2} = \frac{\zeta_1}{\zeta_2}\frac{\rho_1^2}{\rho_2^2}; \tag{3.19}$$

Proof. Please see the Appendix.

Intuitively, a weight assigned on each signal depends on the relative precision and sensitivity to ability of each signal, as suggested by the results in Chapter 2 for the case of information as a scalar. This result also echoes that of Banker and Datar (1989), who show that the optimal weights placed on measures of performance in a strictly moral hazard setting are proportional to the relative precision and sensitivity of the measures. In a moral hazard setting, the measures are contracted on directly, and therefore the relative weights placed on the measures in a linear contract are simply equal to the ratio of their relative effect on PPS. In our setting, the information is not contracted on directly, but rather endogenously shapes the contract menu that is offered, affecting both the salary and bonus of each contract. As a result, it is not the relative effect on the PPS that is equal to the ratio of the precisions and adjusted sensitivities of the pieces of information, but rather the ratio of sensitivity of expected total compensation to the information (which, in a pure moral hazard setting, where salary fundamentally cannot depend on effort, corresponds exactly the relative effects on the PPS).

Proposition 3.1 immediately implies the following corollary, which identifies an important distinction between our results and those that obtain in the strictly moral hazard problem.

Corollary 3.1 *The optimal aggregation of information depends only on the distribution of the information. Thus, it holds for any specification of production function, utility functions of the principal and agent, and for any contract type.*

This corollary suggests that the optimal weights will be identical across some groups of firms. By contrast, Banker and Datar (1989) suggest that the optimal weights on performance measures in a moral hazard setting are typically unique to a given firm (in that they depend, in general, on the optimal value of the action being induced and on the agent's utility function—see Amershi, Banker, and Datar [1990]). This fact can be seen in our model by examining the *H*-type PPS in equation (3.11) (which is equal to the PPS in a pure moral hazard case). This object affects the effort exerted, and it depends on the marginal product of effort (which is normalized to 1 in our model), σ_y^2 (variance of noise from the production function), *c* (the cost of effort), and *R* (risk-aversion).

Clearly, the correct relative weights depend on parameters that are likely to be specific to the agency. This is not the case for the information about ability. The optimal weights placed on information about ability depend only on the distribution of the information in question, and are independent of the specific solution to a given firm, the production technology in question,[37] and the agent's utility function. Fundamentally, this distinction emerges because, in contrast to his effort, the agent does not choose his level of ability during the course of the agency relationship, and nor can he do anything else to

[37] An exception is the case when past production is itself used as information about ability.

affect the distribution of the information. Indeed, the only determinants of the optimal weights are the characteristics of the ability and information distributions, neither of which is affected by the choices made by either the principal or the agent. Thus, the optimal weighting of information will be identical for groups of firms that have a common notion of what "ability" is. This makes intuitive sense especially in the context of executive search. High-level managers are prized for their general managerial ability, which is transferable to other organizations, rather than treated as firm-specific skills. Consistent with this, Custódio, Ferreira, and Matos (2013) find that "generalist CEOs" are paid more than "specialist CEOs."

The properties of the optimal relative weights in Proposition 2 have practical and empirical implications. The value of each piece of information varies positively with its sensitivity and precision. Therefore, the optimal relative weight is also proportional to its sensitivity and precision, consistent with the intuition we obtain from the Proposition 2.4 in Chapter 2. For instance, in the case of CEO recruiting, suppose that the recruiting committee has observed each of the previous two years' earnings of the candidate's current firm; that is, two pieces of information about the candidate's ability (which is an extension of the setting studied by Banker et al. [2013]). If these measures are likely to be equally precise and equally sensitive to the candidate's ability, then they should be given equal weights in forming the set of acceptable contracts. If, on the other hand, the older information is considered to be less precise—if, for instance, ability was thought to evolve over time, so that more recent information is more likely to reflect the present level of ability—then the older information should be given less weight than the more recent information.

Another possible scenario related to past earnings is that earnings quality might differ from one period to another. For example, the incentive to manage earnings might be stronger in one period than other period. In such a case, earnings information for the period with strong earnings management incentives should be given less weight. Suppose that the two available pieces of information about the candidate's ability are the past year's earnings of his firm and the quality of his education. In general, we would not expect these two pieces of information to be equally sensitive to the candidate's level of ability. Suppose further that the quality of the candidate's education is more sensitive to the candidate's ability than is the earnings information. If the recruiting committee considers both to be equally precise, then it should rely more on the education information in designing the set of acceptable compensation contracts. But if the education information is much less precise than the past performance information, then the education information should receive commensurately less weight (and possibly even strictly less weight than the past performance information, depending on their relative precision and sensitivity).

3.5 Conclusion

This paper drew attention to the tradeoffs associated with relying on pre-contracting ability measures in the design of executive compensation schemes. The analytical accounting literature has traditionally emphasized moral hazard in the context of executive compensation and performance measurement, yet managerial ability also potentially plays an important role. Consequently, we argued that further study of adverse selection problems is called upon. We recognized that, in designing the compensation scheme of an executive, the firm has access to noisy measures of the agent's ability pre-contracting. These may include human capital measures (such as the internal review of the executive); as well as some long-term measures that is mainly driven by agent's ability. We formalized this intuition by studying a model in which the principal implements a contract contingent not only on the outcome of interest to the principal but also a noisy signal of the agent's ability.

Our results for the adverse selection setting underscore the value of pre-contract information. Generally, principals will be more willing to incur search costs that inform them about candidates' abilities when the value of information is sufficiently high. An interesting implication of our result that the value of information is quasi-concave in the severity of adverse selection is that the principal might want to engage in search activities that can reduce the type dispersion if the severity is beyond the peak point. While we focused on the acquisition of information that assesses the agent type more accurately, under some conditions the principal will prefer a pool of applicants in which the distribution of the types is not too wide. In many recruiting situations, applicants are pre-sorted based on some clearly and readily observable traits (e.g., the education level and the quality of schools). Such sorting can be viewed as an effort to narrow the type dispersion to a reasonable level (i.e., within the range in which the value of information is increasing) before the principal gathers more information about individual candidates. Our analysis also has an implication regarding whether to search for a new executive internally or externally. Inside candidates offer richer precontract information, while outside candidates might come from a pool of higher-average talents. Engaging a head-hunting firm also changes the pool of available candidates, at a cost.

Principals typically acquire multiple pieces of information about candidates. We find that under a broad class of distributions, pieces of information can be linearly aggregated using optimal relative weights, which are positively related to the precision and sensitivity of the particular pieces of information. These relative weights are reminiscent of the optimal weights in the context of moral hazard, but it is noteworthy that in the context of adverse selection, the weights may be "ability-specific" specific but not "agency-specific". In particular, the optimal weights to be placed on information about ability depend only on the distribution of the information.

CHAPTER 4 OPTIMAL MANAGEMENT CONTROL MECHANISM

4.1 Introduction

The economic theory of the principal-agent model and organizational theory of control have developed in parallel, but they address the related question of the optimal design of a control mechanism. Economic theory relies on a market-based approach, typically using the framework of principal-agent theory, while organizational management theory bases its approach on behavioral and organizational theories. As a matter of fact, various disciplines (e.g., economics, accounting, and marketing) rely on analytical agency models to provide insights on management control, mainly outcome control and effort control. An important omission to this approach is that some agents may have significant other-regarding preference that is explored in the behavioral economics literature. We extend the standard agency framework to incorporate the other-regarding preference and expand the scope of management control mechanisms in accounting to include clan control that is studied in organization theories. This approach in turn allows us to link agency theory to organizational control theory by considering three types of control mechanisms, i.e., outcome control, effort control, and clan control, and identify the importance of outcome measurement, effort measurement, task programmability, and socialization cost in determining the optimal management control mechanism.

The problem of the optimal control structure arises fundamentally from information problems, which have been categorized in the literature in various ways. For our purposes, task noise (or inversely, task programmability) refers to how well the principal or controller knows about the optimal action that she wants the agent to take

(Eisenhardt, 1985) or how well the principal knows about the transformation process (Thompson, 1967)[38]. Outcome noise refers to how well the principal can measure the outcome of interest on which the employee's contingent compensation is based. Finally, effort noise is the difficulty the principal faces in determining which actions the employee has taken (even if the task was specified exactly and the outcome measured precisely).

To illustrate, consider the retail industry. A typical job might include the cashier task, which can probably be easily specified and observed by the principal (who faces little behavior or task noise). The job might also include the salesperson task, which may involve considerable judgment on a per-customer basis, and is therefore much more difficult to specify (due at least to task noise). To take another example, the administration of a business school is likely to face considerable effort noise (professors are monitored infrequently), task noise (it is difficult to prescribe the steps required to produce research), and outcome noise (the quality and impact of the research is often difficult to quantify, at least at the time the researcher must be compensated). These facts are reflected in the complex control structure typical of such an organization.

The organizational theory of control studies how the proper type of control mechanism depends on the particular conditions faced by an organization. Organizational theory posits three forms of control: *outcome control*, i.e. control by measuring outcome; *effort control*, i.e. control by monitoring behavior; and *clan control*, which consists of employing agents whose goals are at least partially aligned with those of the organization, and hence mitigates the goal conflict between the individual and the organization (Ouchi

[38] Eisenhardt (1985) equates the notion of task programmability to the knowledge of transformation process.

1979, 1980, 1981; Eishenhardt 1985, 1989[39]; Govindarajan and Fisher 1990; Kirsch 1996).

The economic theory of the principal-agent model addresses research questions in the design of control mechanism similar to those studied by organization theory, but it employs a different framework and emphasizes on different factors. Agency theory formally models the relationship between a principal, who delegates work, and an agent, who performs that work. When there is a conflict of interest between the two parties, an incentive problem arises. Agency theory describes that relationship using the metaphor of a contract, and aims to find the optimal or second-best contract. Agency theory in accounting highlights the use of information systems and performance measures, and largely focuses on the properties of the pay-performance sensitivities of various performance measures as they relate to the tradeoff between risk-sharing and the provision of incentives (Banker and Datar 1989; Feltham and Xie 1994). Indeed, there is ample evidence that agents are rewarded on the basis of performance measures such as accounting numbers and market returns (e.g., Healy 1985; Sloan 1993; Bushman et al. 1996; Ittner et al. 1997; Core and Guay 1999).

We combine these two approaches, analyzing the management control mechanisms using an analytical principal-agent model with moral hazard and decision delegation under post-contract information asymmetry. In the process, we develop more precise definitions of notions discussed in the extant literature, and we more sharply distinguish them from one another. Our definition of task noise clearly links it to the set

[39] Eisenhardt (1985; 1989) look at the principal-agent model in the perspective of organization control theory. She expands Ouchi's framework to consider the effort noise as an additional determinant of the optimal control mechanism.

of actions that might be performed, as opposed to alternate definitions that relate to the information environment. Outcome noise and effort noise have already been explored in the agency literature—for example, Banker and Datar (1989) show that the relative weight of performance measures should be based on the sensitivity to noise ratio. We show how a similar result relates to the optimal control mechanism. We also establish the concept of clan control in an agency model by introducing an aligned agent—an agent that at least partially shares the same goals as the principal through social interactions unique to the clan control. Such an agent can be employed either by searching or internal training which is called the socialization process. Using this framework, we are able to specify the factors that determine when clan control rather than behavior or outcome control is the optimal organizational strategy. We identify conditions under which the standard Ouchi (1979) findings hold, and we identify important and meaningful deviations from those findings. In particular, our new concept of socialization cost and the interactions among the informational variables, i.e., outcome noise, effort noise, and task noise, give rise to scenarios not previously explored in the literature.

The expanded structure of the principal-agent model also connects to the accounting research on management control systems such as Govindarajan and Fisher (1990), which emphasize the notion of effort noise rather than task noise. They claim that "the determination of outcome reveals behavior and the determination of behavior reveals outcome" or there is a "one-to-one mapping" between outcome and behavior when task can be perfectly specified. We uncover a more nuanced relationship when outcome, behavior, and task noise are explicitly defined. Using our framework, we reformulate the Govindarajan and Fisher model, illuminating both similarities and differences in results.

The rest of the paper proceeds as follows. Section II reviews the prior literature and unifies the terminology. Section III fully develops the analytical model. Section IV presents our main analytical results and discusses economic underpinnings. Finally, Section V concludes.

4.2 BACKGROUND AND LITERATURE REVIEW

4.2.1 Conflict of Interests and Information Asymmetry

In a standard agency framework, there are two key issues that are important in designing and selecting control mechanisms: conflict of interest and information asymmetry. A conflict of interest exists between the owner of an organization and its (non-congruent) employees. Information asymmetry exists regarding the characteristics and outcomes of the complex tasks that the organization performs in pursuing its goals. A traditional accounting system does not generate complete information, and may even fail to generate basic and necessary information. It can also fail to communicate complex ideas and notions. Any control mechanism must deal with these issues, but how they approach these issues could be quite different. The traditional economics approach—also taken by many accounting researchers—focuses on an explicit and enforceable agreement; employment contracts and performance evaluations tend to be based on objective, observable, and therefore contractible metrics. In other words, agents are motivated by extrinsic rewards. Agency models (e.g., Holmstrom 1979) address the first issue by having the principal contract with the agent, specifying compensation based on a performance measure that is fully or partially correlated with the owner's interest; and with the second issue by delegating production-related decisions to the agent to circumvent the information asymmetry problem. Of course, even under a legalistic approach, employees can always communicate directly with the principal to supplement inadequate information that is produced by a formal accounting information system. However, to the extent that such communication is not part of the formal mechanism, it is quite possible that that information is never revealed to the principal because it is not reliable, not verifiable, and therefore not contractible.

4.2.2 Types of Control

We consider the three forms (mechanisms) of organizational control discussed by Ouchi (1979) and others: (1) "outcome control"—control implemented by the observation of some outcome generated by an agent's action; (2) "effort control"—control implemented by the direct observation of the actions undertaken by the agent, and; (3) "clan control"—control implemented by a partial alignment of the goals of the agent and the organization, achieved through a socialization process. Ouchi (1979) identifies two key factors that determine the optimal organizational mechanism: the ability to measure outputs (outcome) and knowledge of the transformation process (lack of which we call "task noise"). While we use the three types of "noises" as a basis of classification, the parlance of organization theory is more varied. However, the basic intuition is identical to the notion of preferring a high signal-to-noise ratio. For example, when task, outcome, and effort noises are low, both outcome control and effort control could be efficient. But if all these noises are high, then it is difficult to motivate and evaluate the agents. In this case, clan control might be a better alternative.

Clearly, none of the three control mechanisms is optimal under all conditions. Both outcome and behavior controls rely on performance measures either directly (behavior) or indirectly (outcome). Which one is better boils down to the efficiency of the information systems in generating the necessary information. These market/contract-based approaches often work well, enabling efficient resource allocation via prices as a synthesizer of information. However, they may not work as well within organizations. The very existence of firms and organizations is telling of the sub-optimality of organizing economic activities solely through a nexus of contracts (Coase 1960). When outcome or effort cannot be measured reliably, clan control might become optimal.

4.2.3 Behavioral Economics and Behavioral Theory

Clan control relies on agents being congruent agents/stewards, whose interests are partially aligned with those of the principals. The traditional neoclassical model of man as the self-interested and rational *homo economicus* was challenged as growing experimental evidence documented clear deviations from predictions, particularly when observing behavior in social interactions between individuals (e.g., Roth et al., 1981; Güth et al., 1982). Behavior in a wide variety of economic games (such as dictator, ultimatum, and third-party punishment games) indicates that people are motivated not only by their own material well-being but also by the economic outcomes experienced by others. In fact, research suggests that only about one-quarter of individuals possess a selfish utility function of the type traditionally assumed within economic models (Bethwaite and Tompkinson, 1996). The determinants of a utility function are instead more complex than is conventionally recognized by economists.

In a traditional utility function, interdependence across individuals is typically ignored. However, Fehr et al. (2007) document that such “other-regarding preferences” can have profound effects on outcomes in markets with moral hazard problems. In these models, individuals can derive disutility both when receiving more than others (i.e., guilt) and when receiving less than others (i.e., envy), motivating a more equal sharing of resources. Fehr and Falk (2002) also provide evidence that non-pecuniary incentives, such as reciprocity, shape human behavior. Because individuals can derive positive utility from repaying the kind and unkind deeds of others, these motivations can augment and interact with economic incentives, and therefore investigating such incentives helps understand economic incentives. In their view, agency theory that typically focuses on individual motives is inadequate as a model of human interactions. Indeed the large literature on social preferences in economics underscores that concerns for the well-being of others cannot be ignored in social interactions (Fehr and Schmidt, 2006).

Experimental evidence has inspired a number of new behavioral theories. Fehr and Schmidt (2006) classify the new behavioral theories into three categories: models of "social preferences" (e.g., Charness and Rabin 2002; Erlei 2008), models of "interdependent preferences" (e.g., Levine 1998; Rotemberg 2008), and models of "intention-based reciprocity" (e.g., Rabin 1993; Dufwenberg and Kirchsteiger 2004). One important component of social preferences theory is altruism (e.g., Andreoni and Miller 2002), which posits that individuals experience "warm glow" and derive greater utility themselves as the utility of others increases. Altruism is a form of unconditional kindness, implying a favor given does not necessarily emerge as a response to a favor received (Andreoni 1989; Andreoni and Miller 2002; Cox et al., 2006; Charness and Rabin 2002). While the pure form of altruism does not require reciprocity, altruism can be interpreted in a broader sense. Behavioral economics literature proposes several types of altruism, including (but not limited to): kin altruism (Hamilton, 1964), reciprocal altruism (Trivers 1971), and strong reciprocity (Gintis 2000; Henrich et al. 2001). For example, strong reciprocity refers to a predisposition to cooperate with others and to punish those who violate the norms of cooperation, at a personal cost (Gintis et al. 2003). Gintis et al. (2003) perform behavioral experiments supporting the strong reciprocity concept and list numerous examples of this phenomenon, such as wage setting by firms (Bewley 2009). Falk and Fischbacher (2006) develop a formal theory of reciprocity that people reward kind actions and punish unkind ones based on economic outcomes, and intention-based models such as Rabin (1993) involve making inferences about how kind or unkind others' acts are based on the choice set from which they made a decision. This rich literature suggests that social considerations can have a powerful and systematic influence on individual behavior.

4.2.4 Clan Control

Clan control takes a behavioral approach to address the issues of conflict of interest and information asymmetry, using an implicit system forged from a socialization process. To achieve this goal, the clan control mechanism requires searching for or training a specific type of agent so that he cares about the owner's interest and nurtures an effective yet informal information system at no additional cost as a by-product of the socialization process.[40] We refer to such a manager as a (goal-) "aligned steward." An aligned steward derives a psychic (non-pecuniary) income from the welfare of the principal or the organization. Relevant information is implicitly conveyed through "rituals, stories, and ceremonies," and other forms of socialization processes and social interactions. And rather than creating or maintaining an explicit information system at

[40] We don't distinguish between the ramifications of searching and training in our paper. For more discussions on the differences, see Ouchi (1979).

some cost, clan control develops its implicit information system as a "natural by-product of social interactions" (Ouchi 1979). We do not, however, assume that goal alignment is necessarily perfect; the cost of socialization increases in the degree of goal alignment and the principal optimizes over the degree of alignment.

Clan control relies on common agreement on a broad range of values and beliefs among the members within the organization. Because it does not rely on any explicit market mechanisms, clan control is perhaps a more demanding mechanism to implement (Ouchi 1979, 1980). However, if successful, clan control could be superior in its ability to deliver profits to the principal and simultaneously convey relevant information relative to explicit information systems. This type of control is thought to be fairly common in professional industries such as hospitals, universities, and other non-profit organizations. Besley and Ghatak (2005) argue that in mission-oriented sectors (e.g., universities), workers are motivated agents who have some non-pecuniary interests in the organization's success such that only low-powered incentive pay is needed. Those agents pursue common goals because they derive intrinsic benefits from doing so. It is also increasingly becoming popular among high-tech and IT companies. For example, the Chairman of Google appears to believe that the best employees are those who do not need much managing and emphasizes the importance of hiring the right people. He says:

> *At Google, we give the impression of not managing the company because we don't really. It sort of has its own borg-like quality if you will. It sort of just moves forward.* [41]

Empirical research in organization and agency literature often abstracts away from clan control because of lack of hard empirical data. However, the notion of clan

[41] From Google Chairman and former CEO Eric Schmidt in 2011.

control has deep theoretical and empirical underpinnings in behavioral economics. Organizational literature identifies which control mechanism is optimal for different circumstances, based on the three types of noises. Table 1 below summarizes our understanding of the literature. Ouchi (1979) focuses on two factors, i.e., outcome noise and task noise. Subsequent studies such as Govindarajan and Fischer (1990) and Kirsch (1996) extend the Ouchi's two-factor control matrix to a three-factor model by adding effort noise. The aims of our model in the next section are; first to make precise the argument in an agency framework and; second, to derive conditions under which each of the three control mechanisms is optimal. We will show that our analysis largely supports the prescriptions of Table 4.1—but not completely. In particular, our analysis suggests that clan control, rather than behavior or outcome control, is optimal for the bottom-right cells.

TABLE 4.1: Summary of Optimal Management Control Mechanisms in the Literature

		Low Task Noise	*High Task Noise*
High Outcome Noise	*High Effort Noise*	*Effort*	*Clan*
	Low Effort Noise	*Effort*	*Effort*
Low Outcome Noise	*Low Effort Noise*	*Outcome or Effort*	*Effort*
	High Effort Noise	*Outcome*	*Outcome*

Finally, before we proceed in setting up our model, we should note that different terminology has sometimes been used to refer to similar concepts in different fields, and likewise, the same terms have been used to refer to distinct concepts in different streams of literature. The table 4.2 summarizes the terminology used in different fields, and relates it to the terminology we will use henceforth in this paper.

Table 4.2: Comparison of Terminology

Economics/Agency Theory	Organization Theory	Our Terminology
Noise in the outcome signal	Outcome measurability/the ability to measure output	Outcome measurement noise
Noise in the outcome production function	N/A[42]	Outcome production noise
Noise in the signal and production function combined	N/A	Outcome noise
Noise in the behavior signal	Behavior measurability	Effort noise
Post-contract information asymmetry on productivity (marginal product of effort)/The principal dictating the level of effort	Task programmability or knowledge of the transformation process	Task noise
To what extent the agent cares about the principal's utility function	To what extent the clan member cares about the controller's goal	Degree of alignment
The market sensitivity to the aligned agent in reservation utility	To what extent is the cost of acquiring/training a clan member	Cost of socialization
Compensation	Reward	Compensation
Performance measure	Outcome/behavior	Outcome signal/behavior signal
Principal	Controller	Principal
Agent	Controllee	Agent
Contracting on the outcome with selecting a self-interested agent	Output control	Outcome control
Contracting on the behavior with selecting a self-interested agent	Effort control	Effort control
Fixed compensation contract with selecting an aligned agent	Clan control	Clan control

[42] The organization theory literature has often confused this type of noise with the task programmability/task noise. We clarify this type of confusion both when formulating our analytical model and interpreting the results.

4.3 THE AGENCY MODEL

A risk-neutral principal (controller) hires a risk-averse agent (controllee) to operate the firm. The agent exerts unobservable effort e, which may in general represent any action undertaken by the agent on behalf of the principal and is the source of the moral hazard problem. The outcome is

$$x(e) = \theta e + \varepsilon_1, \tag{4.1}$$

where $\theta > 0$ is the marginal product of effort, and ε_1 is a normal shock with mean zero and variance $\sigma_{\varepsilon_1}^2 \geq 0$. ε_1 is the *outcome production noise*, representing uncontrollable events that affect the outcome after the agent exerts effort. The outcome per se is not contractible because the principal is not able to observe the true outcome. Instead the principal observes a signal of outcome:

$$y(e) = x(e) + \varepsilon_2, \tag{4.2}$$

where ε_2 is a normal shock with mean zero and variance $\sigma_{\varepsilon_2}^2 \geq 0$. ε_2 is the *outcome measurement noise*, representing the principal's uncertainty at the time when compensation must be paid about the eventual outcome, i.e., some measurement on the true outcome. Without loss of generality, we assume the covariance between outcome production noise and outcome measurement noise is zero. To illustrate the difference between the two sources of uncertainty, we may use firm value as an example. Shareholders don't know the firm's true value which accounts for some economic-wise shocks independent of the manager's action. In practice, stock price, an inherently noisy signal of the firm's true value, is commonly used as a contracting tool to incentivize and compensate the manager.

Conveniently we can rewrite the signal of outcome as

$$y(e) = \theta e + \varepsilon, \tag{4.3}$$

where the additive noise $\varepsilon = \varepsilon_1 + \varepsilon_2$ is *outcome noise*, which includes both the outcome production noise and outcome measurement noise, with mean zero and variance $\sigma_\varepsilon^2 = \sigma_{\varepsilon_1}^2 + \sigma_{\varepsilon_2}^2 \geq 0$. We will henceforth adopt outcome noise to develop the model, but the reader should keep in mind that it is important to distinguish these two different sources of uncertainty, as production noise is sometimes confused with task noise in the organization control literature. We will discuss more about the misunderstandings of task noise and production noise in the results section.

The principal also observes a direct signal of effort

$$z(e) = e + \eta, \tag{4.4}$$

where η is a normal shock with mean zero and variance $\sigma_\eta^2 \geq 0$. This noisy signal of effort, such as information obtained from the accounting information system, can be used to mitigate the moral hazard problem (as in Huddart and Liang 2003). We refer to the variance from this type of random shock as *effort noise*.

It is worthwhile to clarify that the outcome measure is fundamentally different from the effort measure. There are three main differences between the outcome and effort measure in our framework. First, the outcome to effort sensitivity is θ whereas the behavior to effort sensitivity is one. Second, outcome noise, which stems from the uncontrollable events in generating the outcome and observing its signal, is distinct from effort noise, which arises from the difficulty in observing effort. Third, the principal may use either the outcome measure or effort measure as a contracting device, but only the outcome measure is directly related to the principal's interests. In the agency literature, the inverse of the outcome noise $1/\sigma_\varepsilon^2$ is referred to as the precision of the signal of outcome, and the inverse of the effort noise $1/\sigma_\eta^2$ as the precision of the signal of effort

(Banker and Datar, 1989). In the organizational control field, these two objects are referred to as outcome measurability and behavior measurability (Ouchi 1979; Eisenhardt 1985; Kirsch 1996).

To introduce clan control in agency theory, we include two types of agent in our model. The first type is the *self-interested* agent, whose utility function is often assumed in economics models—that is, only the agent's own action and own compensation appear in the agent's objective. The second is the *aligned* agent. The aligned agent cares not only about his own compensation and actions, but also a partial outcome that is of interest to the principal. In addition, the aligned agent reveals the information on task characteristics post-contracting as those types of information is contained in rituals, ceremonies, and other types of social interactions in clan control.[43] Thus, our model allows for clan as a control mechanism by training self-interested agents at some cost—for example, into an aligned agent through some socialization process or simply searching for the right agent.[44] Having introduced the type of aligned agent, we are able to formally construct three forms of control mechanisms within the agency framework: *outcome control* entails contracting on the signal of outcome and hiring a self-interested agent; *effort control* entails contracting on the signal of effort and hiring a self-interested agent; and *clan control* entails employing a self-interested agent and selecting the optimal level of alignment to which the agent will be trained (the agent is then offered a fixed wage

[43] In traditional economic models, truth-telling is costly as in adverse selection models. From the organization control perspective, the revelation of information is just a by-product of social interactions with no additional cost.

[44] From the economics perspective, no practical difference exists between training a aligned type and searching for one. See Ouchi (1979) for the subtle difference stems from social underpinnings.

contract).[45] Apparently, clan control is more demanding with respect to the social underpinnings, but for simplicity we only consider the scenario under which the requirements for outcome, effort, and clan control are all met.

As in Dutta (2008), Feltham and Xie (1994), and Holmstrom and Milgrom (1987), we restrict our analysis to linear compensation contracts.[46] The contract takes the form

$$w_i\big(p_i(e_i)\big) = \alpha_i + \beta_i p_i(e_i), i \in O, B, C \tag{4.5}$$

where $p_i(e)$ are the contracting device used in each forms of control, i.e., $p_O(e) = y\big(x(e)\big)$ for outcome control and $p_B(e) = z(e)$ for effort control; and for clan control, the contract is a fixed wage[47]

$$w_C = \alpha_C. \tag{4.6}$$

Hence, α_O, α_B, and α_C represent the agent's salary in the respective forms of control, β_O the sensitivity of the agent's compensation with respect to the signal of outcome $y\big(x(e)\big)$ in outcome control, and β_B the sensitivity of the agent's compensation with respect to the signal of effort $z(e)$ in effort control.

We use $\lambda \geq 0$ to represent the *degree of alignment* of the agent—the extent to which the agent cares about the outcome of interest to the principal. A self-interested agent is endowed with zero degree of alignment, i.e., $\lambda = 0$ whereas an aligned agent is

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[45] In the behavioral economics literature, many studies suggest that monetary incentives can be harmful when it is mixed with social preferences (Gneezy and Rustichini, 2000; Fehr and Falk, 2002). As a starting point, we model clan control without involving financial incentive to avoid complicating the insights that our model offers.

[46] Linear contract is both common in theoretical agency literature and in practice. Stock option is a common example.

[47] In clan control, β_c is in fact 0.

an agent with any $\lambda > 0$. Upon employed, a self-interested agent can be trained into an aligned agent through some forms of socialization process at certain cost.[48] We assume that the cost of training an aligned agent is quadratic and increasing in λ. Then $r \geq 0$ is the cost of training an aligned agent with the degree of alignment λ, which we refer to as the *socialization cost*. Without loss of generality, we follow the tradition in agency literature by setting up a risk-neutral principal, whose utility function is

$$U_i^P = x(e_i) - w_i\big(p_i(e_i)\big) - r\lambda^2. \tag{4.7}$$

To ensure that the principal's utility maximization problem when she chooses clan control won't reverse to a minimization problem, we make the following assumption.

Assumption 4.1: $r > \frac{\theta^2}{2}$.

We follow the prior literature (Holmstrom 1979, Baiman and Evans 1983, Dye 1983, Banker, Darrough, Li and Threinen 2019) by modeling the agent's utility function as

$$U_i^A = U\Big(w_i\big(p(e_i)\big) - c(e_i, k)\Big), \tag{4.8}$$

where $c(e_i, k)$ is the agent's cost function of exerting effort. In particular, we assume

$$c(e_i, k) = \frac{e_i^2}{2} - ke_i, \tag{4.9}$$

[48] The alternative setup of searching for an aligned agent with higher reservation utility would not change our results.

where k is the *cost characteristic*, a random variable following normal distribution with mean zero and variance $\sigma_k^2 \geq 0$.[49] The cost characteristic is unobservable to both principal and agent pre-contracting, but the distribution of the cost characteristic *F(k)* is common knowledge pre-contracting. The cost characteristic represents the agent's preference for various tasks that he might face after he agrees to take the job. Consider again the retail selling industry. We could imagine that an agent's disutility from spending one full day trying to hustle customers into buying is greater than from spending the same amount of time running the cash register, even for a given level of effort over that time. However, the agent may be uncertain before accepting the job about how his time will be allocated on various tasks, i.e., the agent is likely to suffer more from performing complex tasks than routine and straightforward tasks for the same amount of time spent. We will show that the agent's cost preference over task is essentially the task characteristic studied in the organization control literature in the next section.

The uncertainty about the cost characteristic gives rise to another type of noise, which we call *task noise*. This uncertainty arises because the agent must decide whether to accept the job before knowing what his optimal level of effort, which we call e^*, will be. The task noise $Var(e^*)$ can be interpreted as indicating how well the agent's optimal level of effort is known *ex ante*. It is important to distinguish between effort noise and task noise, which are related but distinct concepts. Whereas effort noise is a feature of the information available to the principal about the agent's action, task uncertainty is a

[49] We assume cost characteristic is not correlated with outcome noise and effort noise without loss of generality. Detailed discussions on the normality assumption are covered in the appendix and simulation results are presented for a variety of distributions.

characteristic of the job itself, and represents the principal's degree of familiarity with the production process itself. It is important to note that in the presence of task noise, the principal may not be able to evaluate and reward the agent even if the agent's action can be accurately observed because the principal will remain uncertain about the productivity of the chosen action (Kirsch, 1996).

To better understand the uncertainty objects defined above, consider again the example of an academic research setting. In research universities, faculty are rewarded based on the number of publications in top tier academic journals, which is the outcome observed by the administration. However, conducting research is a relatively complex job—in our terminology, the task uncertainty of doing research is high. There are no routines to follow, and indeed a given set of activities in doing research might lead to different results for different faculty. The administration does not observe every action undertaken by the researcher to accomplish the task, and might observe only a weak signal of the researcher's effort, such as hours spent on research or the hours communicating with colleagues—in other words, the effort noise is high. And while the quality of research output is positively correlated to effort, there is considerable noise in the production function—that is, there is high production uncertainty. Finally, even after publication, the ultimate impact of a paper will not be evident for some years—there is high outcome measurement uncertainty. Together, these last two imply high outcome noise.

After entering the contract and starting performing the task, the agent privately learns about his cost preference over the task, i.e., the cost characteristic k. Therefore, information asymmetry on the cost characteristic arises post-contracting. However, note the difference between the post-contracting information asymmetry setting in our model and a pre-contracting information asymmetry setting which is known as adverse selection. In our model, there is no adverse selection problem because at the time of contracting, both the agent and the principal are uninformed with respect to the cost characteristic. Thus, ours is a model of only moral hazard but with post-contracting information asymmetry. Examples of post-contracting information models include Dye (1983) and Banker and Data (1990).

As is common in the agency literature (Dutta 2008; Feltham and Xie 1994), we assume the agent has constant absolute risk aversion (CARA) preferences with coefficient of absolute risk aversion R and negative exponential utility function

$$U_i^A = U(w_i(p_i(e)) - c(e_i)) = -exp\{-R[w_i(p_i(e)) - c(e_i)]\}. \qquad (4.10)$$

Figure 4.1: Information Structure for Outcome/Effort control[50]

Both the principal and agent don't observe the cost/task characteristic. (Task uncertainty to both P and A)	The agent privately learns the cost/task characteristic post-contract and chooses the optimal level of effort based such information. (Task uncertainty to P)	Then the agent exerts effort and produces the outcome with random state of nature. (Outcome production uncertainty to both P and A)	The principal may choose to observe a noisy signal on the agent's behavior. (Effort noise to both P and A)	Both the principal and agent observe a noisy signal on the outcome. (Outcome measurement uncertainty to both P and A)

[50] P and A are notations for principal and agent, respectively.

Figure 4.2: Information Structure for Clan Control

Both the principal and agent don't observe the cost/task characteristic. (Task noise to both P and A)	The principal trains the self-interested agent into an aligned agent at the optimal level of socialization cost.	The agent privately learns the cost/task characteristic post-contract.	Then the agent exerts effort and produces the outcome with random state of nature. (Outcome production noise to both P and A)	The cost/task characteristic is revealed to the principal. (Outcome measurement noise to both P and A)

The information structure of our model is shown in Figure 4.1 and 4.2. Another important distinction between the traditional moral hazard model and our model is the agent's decision rule. In the traditional agency literature, the agent's effort/action is dictated by the principal through the corresponding mechanism design by which the principal elicits the optimal level of effort through enforcing the incentive compatibility constraint. In such scenario, the optimal level of effort is known by the principal. However, in our model (outcome/effort control), the principal is not capable of determining the exact optimal level of effort due to unobservable cost/task characteristic. We adopt the decision delegation setting to the extent that the agent's effort choice is fully delegated to the agent himself instead of being elicited by the principal.[51] In other words, the agent privately chooses the optimal level of effort based on private new information. This feature of our model is similar to the participative management control system as in Baiman and Evans (1983) where the agent possesses better pre-decision information than the principal. Raith (2005) also considers the decision delegation when

[51] Decision delegation is preferable by the principal as the agent could utilize the task related information to improve production.

the task related information is private to investigate the trade-off between output-based pay and behavior-based pay. We will demonstrate in details in the results section that the principal will benefit from such mechanism because the agent can take advantage of the task related information that is only observable to him. The Raith (2005) model also possesses this feature.

To find the optimal contract, we first derive the optimal compensation contract for each control mechanism. By using backward induction, the principal chooses the control mechanism that delivers her the highest expected utility.

The principal's problem under outcome/effort control is the following:

$$\max_{\alpha_i,\beta_i,e_i} E\left[y(e_i) - w_i\left(p_i(e_i,k)\right)\right], \tag{4.11}$$

subjects to the individual rationality (IR) constraint[52]

$$\int_{-\infty}^{\infty} E\left[U\left(w_i\left(p_i(e_i)\right)\right) - c(e_i,k)\right] dF(k) \geq U(r_0), \tag{4.12}$$

and the incentive compatibility (IC) constraint

$$e_i = \underset{\hat{e}_i}{argmax} E\left[U\left(w_i\left(p_i(e_i)\right), \hat{e}_i\right) - c(\hat{e}_i,k)\right]. \tag{4.13}$$

The expectations in the IR and IC constraint are taken with respect to outcome noise or effort noise depending on the chosen control system. Notice that in the IC constraint, the agent chooses the optimal level of effort based on the privately observed cost characteristic that the agent observes only after entering the contract, whereas in the IR constraint the principal needs to take expectations with respect to both the cost characteristic and the noise in the signals. It is worthwhile to point that the expression

[52] The reservation utility is normalized to 0 without loss of generality in the following analysis.

$E[U(w_i(p_i(e_i)), e_i) - c(e_i, k)]$ corresponds to CE_A^{Pre} in equation (4.13) and $\int_{-\infty}^{\infty} E[U(w_i(p_i(e_i)), e_i) - c(e_i, k)]dF(k)$ corresponds to CE_A^{Post} in equation (4.12). The information structure on cost/task characteristic and the mechanism of the contract are the keys to understand the difference between CE_A^{Pre} and CE_A^{Post}. Specifically, when the principal designs the incentive compatibility constraint for the agent, she let the agent make his own choice on the level of effort and therefore the information on cost/task characteristics is available to the agent when he makes his move. However, when the principal designs the individual rationality constraint for the agent, she must calculate the expected value with respect to signal noise as well as the noise on cost/task characteristic to ensure participation since both the principal and the agent do not observe such information pre-contracting.

The principal's problem under clan control is the following:

$$\max_{\alpha_C, e_C, \lambda} E[y(x(e_C)) - \alpha_C - r\lambda^2], \tag{4.14}$$

subjects to the individual rationality (IR) constraint

$$E[U(\alpha_C - c(e_C, k))] \geq U(r_0) \text{ for all } k, \tag{4.15}$$

and the incentive compatibility (IC) constraint

$$e_C = \underset{\hat{e}_C}{argmax} E[U(\alpha_C, \hat{e}_C) - c(\hat{e}_C)]. \tag{4.16}$$

The clan control system guarantees that the aligned agent reveals his private information on cost/task characteristic through social interaction such as rituals, stories, and ceremonies, which is a by-product of the social training process (Ouchi, 1979). Solutions to the principal's problems in each control mechanism in the appendix characterize the agent's optimal level of effort, the optimal pay-performance sensitivity, the optimal degree of alignment, and the principal's expected utility.

4.4 The Results

In this section, we present our main findings. Solutions to the model, characteristics of the solution, and related discussion can be found in the Appendix.

LEMMA 4.1 (Task Uncertainty): *For all the three types of control mechanism, the task noise is*

$$Var(\hat{e}_i) = \sigma_k^2. \tag{4.17}$$

PROOF: Please see the Appendix.

Lemma 1 states that the variance of the agent's optimal level of effort is identical to the variance of the cost/task characteristic, irrespective of the type of control implemented. This is the main reason that cost/task characteristic can be used interchangeably in our analysis. Recall that in the previous section we define task noise as the variance of the agent's optimal level of effort. Therefore, we will henceforth refer to σ_k^2 as the task noise. Also recall that the task noise can be interpreted as how well the principal knows about the agent's optimal level of effort. When the task can be clearly specified or easily programmed (alternatively speaking, the distribution of task characteristic degenerates to a fixed point), the principal can dictate the optimal level of effort the same way as in traditional moral hazard models. However, when the task is complicated and difficult to program, the principal would not be certain about the agent's optimal level effort and thus delegates the choice of effort to the agent. Lemma 1 underscores the fact that task noise is fundamentally a feature of the task itself, not of the type of control system used.

The following lemma derives the principal's expected utility at the optimum for outcome control, effort control, and clan control, respectively.

LEMMA 4.2 (Principal's Expected Utility): *Suppose the principal implements outcome control such that she hires a self-interested agent and contracts on the signal of outcome. Then the principal's expected utility at the optimum is*

$$\pi_O = \frac{\theta^2}{2\left(1+\frac{R\sigma_k^2}{1+R\sigma_k^2}+\frac{R\sigma_\varepsilon^2}{\theta^2}\right)}+\frac{Log(1+R\sigma_k^2)}{2R}. \tag{4.18}$$

Suppose the principal implements effort control such that she hires a self-interested agent and contracts on the signal of effort. Then the principal's expected utility at the optimum is

$$\pi_B = \frac{\theta^2}{2\left(1+\frac{R\sigma_k^2}{1+R\sigma_k^2}+R\sigma_\eta^2\right)}+\frac{Log(1+R\sigma_k^2)}{2R}. \tag{4.19}$$

Suppose the principal implements clan control such that she trains the self-interested agent into an aligned agent, selects the optimal degree of alignment, and offers the agent a fixed wage. Then the principal's expected utility at the optimum is

$$\pi_C = \frac{\theta^2}{2\left(\frac{2r-2\theta^2}{\theta^2}+1+\frac{R\sigma_\varepsilon^2}{\theta^2}\right)}+\frac{\sigma_k^2}{2}. \tag{4.20}$$

PROOF: Please see the Appendix.

Note that the principal's expected utility at the optimum from using outcome control is very similar to that from using effort control. The only difference between equations (4.18) and (4.19) is the expression $\frac{R\sigma_\varepsilon^2}{\theta^2}$ in outcome control and $R\sigma_\eta^2$ in effort control. This result is intuitive if we scale the signal of effort in equation (4.4) as

$$z(e)\cdot\theta = \theta e + \theta\eta. \tag{4.21}$$

Obviously, the signal of effort can be alternatively viewed as another outcome signal by multiplying marginal product of effort. The variance of this artificial outcome signal is then $Var(\theta\eta) = \theta^2\sigma_\eta^2$, which confirms our results as in equations (4.18) and (4.19). Furthermore, by setting σ_k^2 to 0, equation (4.18) becomes

$$\pi_O = \frac{\theta^2}{2\left(1+\frac{R\sigma_\varepsilon^2}{\theta^2}\right)}. \tag{4.22}$$

Equation (4.22) exactly coincides with the solution when task characteristic is common knowledge as in traditional moral hazard models (apparently this also holds for the effort control scenario). Clan control is quite different from the other two control systems. The term $\frac{2r-2\theta^2}{\theta^2}$ in the denominator reflects the relative cost of intrinsically motivating an agent (socialization) to extrinsically incentivizing an agent (incentive compensation), since *r* is the cost of implementing socialization and θ^2 measures the cost of implementing incentive contract. Therefore, there are trade-offs between the relative cost of socialization and risk sharing in the first term of equation (4.18).

To better understand the economic insights on the principal's expected utility at the optimum from each type of control, we need to take a closer look at the agent's individual rationality constraint. We take the IR constraint of the outcome control as an example and similar analysis can be done for the other two controls,

$$\alpha_O - \frac{R\beta_O^2\sigma_\varepsilon^2}{2} + \frac{\theta^2\beta_O^2}{2(1+R\sigma_k^2)} + \frac{\mathrm{Log}(1+R\sigma_k^2)}{2R} = r_0. \tag{4.23}$$

We can rewrite the LHS of the equation (4.23) as follows,

$$\underbrace{\alpha_O + \theta^2\beta_O^2}_{\text{(1)risk neutral compensation}} - \underbrace{\frac{\theta^2\beta_O^2}{2}}_{\text{(2)risk neutral cost of effort}} - \underbrace{\frac{\beta_O^2}{2}R\sigma_\varepsilon^2}_{\text{(3)risk aversion to outcome noise}} - \underbrace{\frac{\theta^2\beta_O^2R\sigma_k^2}{2(1+R\sigma_k^2)}}_{\text{(4)risk aversion to task noise}} + \underbrace{\frac{\mathrm{Log}(1+R\sigma_k^2)}{2R}}_{\text{(5)Effect of decision delegation}}. \tag{4.24}$$

The LHS of equation (42.3) corresponds to the agent's certainty equivalent pre-contracting, $\mathrm{CE}_A^{\mathrm{Pre}}$. As one can see, our framework intuitively breaks down the agent's expected utility into linearly additive components that have important economic underpinnings. The first three terms can be intuitively explained by the mean-variance property of the CARA-Exponential utility function. The last two terms are the most interesting part, as they are not found in other moral hazard models. Intuitively, the positive fourth term in equation (4.24) is decreasing in task noise, reflecting the agent's risk aversion with respect to task noise. This term is not in the same fashion as the third term because task noise influences both effort level and the contingent part of the compensation. The fifth term is the benefit of decision delegation stemming from the mechanism that the agent chooses his effort conditional on the observed task characteristics. If the agent is not able to learn the realized task/cost characteristics post-contracting, then the corresponding certainty equivalent is decreasing in task noise, lacking the increasing fifth term in equation (4.24). We can therefore define the *effect of delegation* as the increasing term (w/r/t task noise) in equation (4.24), which is independent of PPS. And we can technically prove that this effect of delegation is always *positive*. Technically this interesting result stems from Jensen's inequality and the convexity of point-wise maximization with respect to effort and the result is true in general, in spite of the specifications for the utility, outcome, and compensation forms. It is intuitive to see that the effect of delegation is increasing in task uncertainty as it is more "beneficial" for the agent to learn about the true task characteristics from an ambiguous prior belief. However, the agent is actually hurt by such effect of delegation since he can't quit once the contract is signed and therefore the principal is able to extract such benefits from him. More analysis on this effect of delegation from the principal's side is covered in the next lemma.

Gaining insights from the analysis above, we characterize the benefits of delegation to the principal among the three types of control.

LEMMA 4.3 (Benefit of Decision Delegation): *Define the benefit of decision delegation as to the extent which the principal can directly extract from the agent's salary due to* *decision delegation. Then the benefit of delegation in three forms of control is increasing in task noise and the benefit under clan control is higher.*

$$Benefit_O = Benefit_B \leq Benefit_C.$$

PROOF: Please see the Appendix.

Lemma 4.3 further looks at the part (5) of equation (4.24) that helps to draw insights on the difference among the principal's payoff under three forms of control. Equation (4.24) is about the agent's expected utility before the contract is signed and this information is known to both parties. In designing the contract, the principal only offers the agent the compensation to the amount that the agent's expected utility is not different from his outside option. As a result, the principal will pay the agent a risk aversion premium and extract the effects of delegation which is of the main focus of Lemma 4.3. Furthermore, the effect of delegation in each forms of control is independent of the agent's pay-performance sensitivity (PPS). Lemma 4.3 is one of the keys to understand the economic trade-off among the tree types of control. Abstracting away from other factors, clan control is better than the other two control types simply because the principal can make the best use of decision delegation under the clan system, which is not surprising as an aligned agent is more beneficial to the principal.

The effects of outcome noise and effort noise simply follows the intuition from traditional agency models. We next perform basic comparative statics on the principal's expected utility with respect to task noise.

LEMMA 4.4 (U-Shape and Monotonicity):

The effect of task noise on the principal's expected utility under clan control exhibits monotone increasing.

The effect of task noise on the principal's expected utility under outcome control exhibits U-shape (monotone increasing), when outcome noise is low (high),

The effect of task noise on the principal's expected utility under effort control exhibits U-shape (monotone increasing), when effort noise is low (high),

PROOF: Please see the Appendix.

The impact of task noise on the principal's expected utility under clan control only stems from the benefit of delegation, and hence positive, because cost/task characteristic is revealed to the principal via the socialization process in the clan system. In turn, the agent is forced with less compensation when the task uncertainty is high, which ultimately benefits the principal. The last two series of comparative statics predict a possible U-shape scenario and a monotone scenario. Take outcome control for example. As discussed in details for Lemma 4.2 and 4.3, there are competing effects for task noise on the principal's expected utility under outcome/effort control. First, there is risk aversion to such noisy information that comes from the agent's risk attitude. Second, there is positive value of decision delegation due to the more informed agent. Lemma 4.4 predicts that in certain scenarios, the principal's expected utility is first decreasing and then increasing in task noise, thus presenting a U-shape. This is intuitive as the risk aversion effect outweighs the delegation effect when task noise is low. However, the delegation effect increases more and eventually outweighs the risk aversion effect as task noise increases. In addition, Lemma 4.4 also predicts a monotonic increasing pattern for the effect of task uncertainty. This is also intuitive as delegation is more useful when outcome/effort noise is higher.

We next perform a preliminary comparison between the principal's expected utility at the optimum under each forms of control and explore the underlying economic insights, aiming to provide an economic foundation to understand our analytical results on the control matrix. Lemma 4.5 below compares outcome control and effort control.

LEMMA 4.5 (Behavior versus Outcome): *If the outcome noise has the same magnitude as the effort noise, then*

- *the principal's expected utility from using outcome control is larger than her expected utility from using effort control if and only if* $\theta \geq 1$.

- *the principal's expected utility from using outcome control is smaller than her expected utility from using effort control if and only if* $\theta \leq 1$.

PROOF: Proof follows by examination of equations (4.18) and (4.19).

Lemma 4.5 can be explained in terms of the sensitivity to noise ratio as in Banker and Datar (1989). The sensitivity to noise ratio of the signal of outcome is θ/σ whereas the sensitivity to noise ratio of the signal of effort is $1/\sigma$. If the productivity θ is larger than 1, then the sensitivity to noise ratio for outcome control is larger than the sensitivity to noise ratio for effort control. In that case, the value of using outcome control is larger than effort control. Lemma 4.5 shows that the trade-off between outcome control and effort control depends solely on the degree of (normalized) uncertainty of the signals, so that outcome control is better than effort control when the outcome signal is relatively less noisy than the behavior signal, and vice versa. An implication is that we can, without loss of generality, limit our attention to studying how outcome noise and task noise affect the choice of optimal control mechanism by standardizing the effort noise.

Lemma 4.6 below compares clan control and outcome control. By virtue of Lemma 4.5, the comparison between clan control and effort control is trivial.

LEMMA 4.6 (Clan vs Outcome):

- *If* $r < \theta^2$*, then the clan control mechanism strictly dominates the outcome control mechanism.*
- *If* $r = \theta^2$*, then* $\pi^{CLAN} = \pi^{OUTCOME}$ *iff* $\sigma_k^2 = 0$ *and* $\pi^{CLAN} > \pi^{OUTCOME}$ *for all* $\sigma_k^2 > 0$.
- *If* $r > \theta^2$*, then there exists a unique* $\sigma_k^2 > 0$ *such that* $\pi^{CLAN} = \pi^{OUTCOME}$.

PROOF: Please see the Appendix.

Comparing equation (4.18) and (4.19) will immediately present two different parts between the principal's expected utility under the two control mechanisms. The most obvious one is the last term in the utility equations. As shown in Lemma 3, $\frac{\text{Log}(1+R\sigma_k^2)}{2R}$ is referred to as the benefit of delegation under outcome/effort control, whereas $\frac{\sigma_k^2}{2}$ is referred to as the benefit of delegation under clan control. In addition, Lemma 4.3 says that the latter is higher.

Before proceeding to discuss the results of Lemma 4.6, two fundamentally different aspects between the outcome and clan control mechanism should be pointed out. First, in implementing outcome control, the principal provides monetary incentive to the agent such that a proportion of the outcome is extrinsically aligned to the agent's interest through contingent pay; and the marginal cost of providing such incentive is θ^2 as explained earlier. However, in implementing clan control, the principal elicits intrinsic motivation by socializing the self-interested agent into an aligned one; and the cost of doing so is the socialization cost$r$. Second, the effort choice is delegated to the agent since the principal is not able to dictate the level of optimal effort in the case of outcome control, and consequently the principal benefits from such delegation scheme by having the more informed agent to make decision conditional on the task characteristic that is unknown to the principal throughout the whole game. However, in clan control, the information on task characteristic is revealed to the principal via the socialization process, and therefore the principal faces a "First-Best" problem with respect to task noise.[53]

Using the insights above, we could present a preliminary comparison between outcome control and clan control. Lemma 6 states that in trading off the control mechanism one crucial factor is the socialization cost, which has not been studied in the organization control literature (Ouchi 1979; Eisenhardt 1985, 1989; Govindarajan and Fisher 1990). If the socialization cost (the cost of intrinsically motivating the agent, in

[53] The optimization problem is still second best due to moral hazard, but there is no private information with respect to cost/task characteristic.

economic perspective) is smaller than the cost of extrinsically providing incentive to the agent ($r < \theta^2$), then the denominator of the first term in equation (4.20) is smaller than that in equation (4.18), leading to the obvious conclusion that clan control is better than outcome control. Suppose it is equal costly to implement the monetary incentive and socialization, the principal would be indifferent between clan and outcome control if and only if the benefit of delegation is the same under the two control modes, which is achieved under perfect information on task characteristic. Finally, a more general yet interesting scenario arises when the socialization cost is higher than the cost of incentive provision. We argue that this is most likely the real-world scenarios as outcome control would be extinct if the cost of socialization is a cost-effective approach. In this scenario, the principal trades off the incremental cost of adopting the socialization mechanism with respect to traditional incentive provision with the benefit of reducing the post-contract information asymmetry. In particular, we have proved in the appendix that the outcome control is better when task noise is smaller and clan control is better when task noise is larger, simply because the benefit of addressing the post-contract information asymmetry through socialization increases relatively faster when cost/task characteristic is noisier.

In the next part of this subsection, we present the main results with respect to the control matrix. Consider the border line in Ouchi's control matrix as the indifference curve, and then by studying the nexus points and shape of the indifference curve we are able to draw insights on the general trade-off under the three control modes. Theorem 4.1 formally proves the existence of the indifference point in the control matrix. Figure 4.3 is generated using Monte-Carlo simulation.

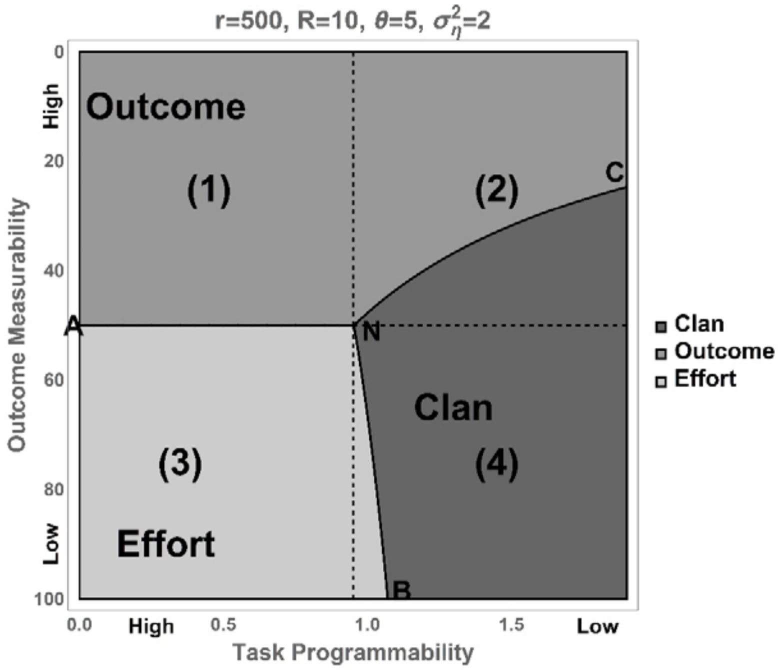
THEOREM 4.1 (Conditions for the Indifference Nexus Point):

Case 1: If the cost of implementing socialization is higher than the cost of extrinsically providing incentive compensation, there exists one unique pair of combination (task noise, outcome noise), i.e., the indifference point, to the extent that the principal is indifferent in choosing the three types of control, outcome, effort, or clan. In other words, the three indifference curve joins at one point.

Case 2: If the cost of implementing socialization is lower than the cost of extrinsically providing incentive compensation, then there are unlimited pair of combination (task noise, outcome noise) such that the principal is indifferent from using effort and clan control, both of which dominate the outcome control.

PROOF: Please see the Appendix.

Figure 4.3: Existence of Nexus Point



Recall that we have standardized effort noise by virtue of Lemma 4.5. Though effort noise is not reflected in Figure 4.3, it still plays an important role in formulating the control matrix. Theorem 4.1 reveals that the optimal strategy of control is fundamentally a function of socialization cost, outcome noise, effort noise, and task noise. Ouchi (1979) argues that the optimal strategy of control is determined by two factors, i.e., outcome noise and task noise in our terms. Later studies such as Govindarajan and Fischer (1990) and Kirsch (1996) extend Ouchi's two factors control matrix to a three factors model by adding effort noise. However, neither do the studies in the literature highlight the crucial role of the socialization cost in determining the optimal strategy of control, nor do they distinguish the two closely related factors, effort noise and task noise. Furthermore, Theorem 4.1 along with Lemma 4.5 and 4.6 show the different roles those four dimensions involved in choosing the optimal strategy of control. For example, effort noise only matters in the context when a comparison involves effort control.

Theorem 4.2 characterizes the pattern of the indifference curve in our control matrix. Our control matrix can be simply illustrated using Figure 4.3 above, as it characterizes all possible scenarios.

THEOREM 4.2 (Characteristic of the Indifference Curve):

- *In the first case of Theorem 4.1, the indifference curve between outcome and effort control OA is horizontal, between behavior and clan control OB is upward sloping, and between outcome and clan control OC is downward sloping.*
- *In the second case of Theorem 4.1, the indifference curve between effort control and clan control is upward sloping.*

PROOF: Please see the Appendix.

Theorem 4.2 analytically derives the pattern of the control matrix in our model corresponding to the Ouchi (1979) setting. The contour line of the control matrix with respect to the two dimensions, task noise and outcome noise, always behave as the pattern shown in Figure 4.3 above such that, OA is a horizontally straight line (degenerates to point A in case 2), OB is a upward sloping curve, and OC is a downward sloping curve (C degenerates to origin point in case 2). Ouchi (1979)'s model presents straight-line cross type cutoffs on the two dimensions, outcome noise and task noise, missing the possibility of interactions between them. For instance, Ouchi predicts that clan control should be used when both task noise and outcome noise are high. However, our model predicts that, effort control is possible to be optimal under the same circumstances, the result of which relies on the relative magnitude between outcome noise and task noise. Furthermore, the control matrix in the organization literature is dichotomous, though the intuition can be carried over to continuum case, the exact pattern of the continuum case might be different. The new insight here is that one might also need to consider the relative relationship between outcome noise and task noise in choosing the optimal strategy of control, not simply implementing some absolute threshold rules. Indeed, the absolute threshold classification as in Ouchi's 2x2 matrix has many great features such as convenience and free of ambiguity, we will expand and reformulate Ouchi's original 2x2 matrix using our new results at the end of the analysis.

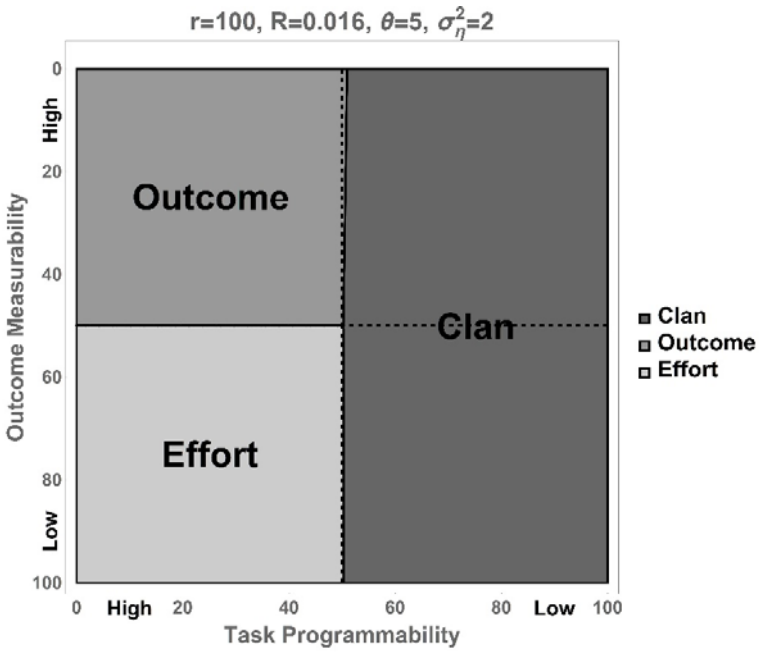
COROLLARY 4.1 (Risk-Neutrality): *Suppose the agent is risk-neutral, the control matrix exhibits straight-line indifference curves.*
PROOF: Please see the Appendix.

Figure 4.4 presents an extreme simulation result when the agent is very close to risk neutral.[54] One can see that the indifference curve becomes almost similar to Ouchi's original results except one major difference pointed out in Theorem 4.3. The organization control literature is not particularly concerned with the controller/comtrollee's risk attitude and the deduction of optimal control matrix in organization control theory is independent of risk concerns. Therefore, we argue that while our results extend Ouchi's framework to a broader scope with risk attitudes, the almost risk-neutral case as in Figure 4.4 is a direct analogy of Ouchi's setting in our model. This implies that even if we mimic Ouchi's settings completely, there is still a major difference between the analytical

[54] As discussed in the previous lemma and Theorem 4.1, the socialization cost is picked at a relatively higher value to prevent obtaining the degenerate Case 2 scenario as in Theorem 4.1.

results and the Ouchi's, highlighting new insights on the effect of decision delegation and trade-off between socialization and monetary incentive provision as discussed in the previous analysis. In Figure 4.4, the bottom-right part is pure clan control instead of outcome control as in Ouchi's results. Such difference highlights the crucial role of task noise in choosing the optimal control mechanism. When task noise is sufficiently large, the principal would find only clan control appealing in spite of the magnitude of outcome noise and effort noise.

Figure 4.4: Risk Neutrality



The following results describe how the nexus point O behaves as other agency parameters change and how our control matrix changes with respect to risk attitude.

COROLLARY 4.2:

- *As the socialization cost increases, the nexus point O starts to move from left to right horizontally.*
- *As the productivity increases, the nexus point O starts to move from bottom to top vertically.*

- *As the agent become more risk-averse, the indifference curves rotate clockwise and the curvature of the curve increase.*

PROOF: Please see the Appendix.

The above results are intuitive. Clan control is less appealing when socialization cost is relatively higher, exhibiting less clan area in our control matrix. Holding the effort noise constant, the outcome signal becomes relatively more precise when the marginal product of effort increases, and thus more outcome area in our control matrix. This explanation also echoes with Banker and Datar's (1989) sensitivity-precision results. Furthermore, the principal trades off between the optimal-risk sharing and the benefit of decision delegation and therefore more risk-averse agent implies less clan area in the control matrix.

4.5 DISCUSSION

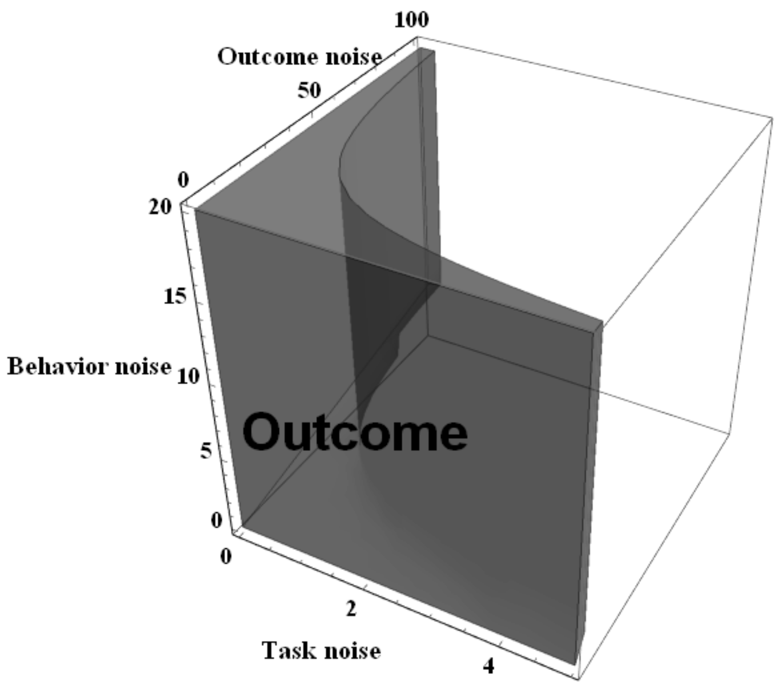
We next reformulate the Ouchi's original 2x2 control matrix using the analytical results and insights gained from Theorem 4.1, 4.2, and 4.3. As one can see, our analytical results are exclusive, for example, case 2 in Theorem 4.1 is rare but still possible in reality.[55] In the next corollary we will focus on both cases in Theorem 4.1 and provide sensible examples in presenting our version of the control matrix. Figure 4.5 in the appendix provides an example of the choice of control mechanism in three dimensions.

Because Ouchi (1979)'s framework is dichotomous and conceptual, the exact quantification of high/low noise (or in Ouchi's words, perfect/imperfect measurement) is not specified. So, one would probably argue that the Ouchi's control matrix is founded on extreme values of uncertainties, i.e., almost 0 for low uncertainty and sufficiently large

[55] As an example, NPOs nowadays increasingly use online technology to recruit members, rendering the socialization cost significantly lower, and consequently the clan control is indeed observed commonly in NPOs.

for high uncertainty. Following this argument, we derive the extreme case scenario in Corollary 4.3 below.

Figure 4.5: Outcome Control in the Control Space



COROLLARY 4.3 (Extreme Dichotomy):

Table 4.3: Extreme Values of Uncertainties

Task / Outcome	*Extremely Low*	*Extremely High*
Extremely High	*Behavior*	*Clan*
Extremely Low	*Outcome*	*Clan*

PROOF: Proof is omitted.

Corollary 4.3 has some practical implications, for example, the principal could rule out the possibility of clan control if the task is simple and straightforward and he can almost be sure to choose clan control if the task is extremely complicated. The above results provide a simple general rule under certain extreme conditions. If the task is simple, for instance, the assembly worker of the Iphone assembly line in the Foxconn factory, it is then almost certain for the manager to rule out the possibility of clan control in choosing the control scheme[56]. However, if the task is extremely complicated, for instance, a software engineer at Google, whose job responsibility involves develop new software and make innovations, then clan control should be chosen without even considering other factors. This is what Google actually does when it hires new software engineers as we have mentioned in the introduction part.

A more fundamental difference arises when outcome measurability is extremely high and task programmability is extremely low (bottom-right cell of Table 4.3). Ouchi (1979) suggests that outcome control is optimal in such case, but our analysis prefers clan control. Even if task noise is not extremely high, we have proved in Theorem 4.3 that the indifference curve between clan and outcome is concave for relatively large task uncertainty, suggesting more clan in such scenario. Indeed, we find that in such a case, outcome control is dominated by clan control even if the principal can perfectly observe the outcome because the task is hard to specify and the principal would find implementing clan control beneficial in reducing the information asymmetry arisen directly post-contracting. Such situations are actually common in practice. For instance, a university may count the number of publications produced by a researcher, or may use a citation index to measure a researcher's output. In either case, output is rather accurately measured (i.e., outcome noise is extremely low). At the same time, effort measurability is quite low for researchers—there are typically few restrictions on time spent in the office or other required tasks. Recall that we are fixating effort noise in our formal analysis, so this low effort measurability and high outcome measurability scenario seems to be more

[56] As a matter of fact, Foxconn has already replaced a portion of its assembly worker with robots.

favorable in using outcome control if one follows Ouchi's argument. Finally, task programmability is extremely low for conducting research related activities such as doing exploratory lab experiments, writing new research papers, etc. It is generally not possible to specify a routine or set of steps that can be followed to produce research without getting hands on the research first—the specific research task is difficult to program from the university administrative perspective. Once a researcher is hired and starts to work, for instance, an accounting researcher would be better informed about how complex the task is given his early involvement with the research itself, the research environment of the department, the funding, etc. In the terms of agency theory, the researcher possesses private information after he is hired. Using Ouchi (1979)'s control matrix, outcome control is no doubt the optimal mechanism as high outcome measurability and low task programmability exactly corresponds the criterion for outcome control. But if we look at the real world, a researcher's compensation is typically salary-based. Furthermore, universities only hire candidates with an intrinsic interest in conducting research—in our terminology, aligned agents. Indeed, one of the roles of PhD education is to train the potential researchers to become intrinsically motivated to conducting research—in the terminology of control literature, the socialization process. If the university comes to believe that a researcher is not an aligned type, then the response of administration is not usually to add incentives to the researcher's contract, but rather to terminate employment. Pearce et al. (1985) gives another example, finding that the implementation of outcome control on managers had no significant effects on organizational performance. They argue that this result could arise because (1) the nature of the managerial work is too complex, or (2) managers have limited control over organizational performance. In our terminology, task programmability is low and therefore outcome control is suboptimal.

It is also useful to compare our results to those of Govindarajan and Fisher (1990), which also incorporated outcome, effort, and task noise. Here we only demonstrate the case of high socialization cost when we select extreme dichotomous values for outcome/effort/task noise, since similar analysis can be applied to the low socialization cost case and non-extreme values.

TABLE 4.4: Govindarajan and Fisher (1990)

		Low Task Noise	High Task Noise
Low Outcome Noise	Low Effort Noise	Outcome or Effort	Outcome
	High Effort Noise	Outcome	Outcome
High Outcome Noise	Low Effort Noise	Effort	Effort
	High Effort Noise	Effort	Effort

TABLE 4.5: Reformulation of Govindarajan and Fisher (High Socialization Cost)

		Low Task Noise	High Task Noise
Low Outcome Noise	Low Effort Noise	Outcome or Effort	Clan
	High Effort noise	Outcome	Clan
High Outcome Noise	Low Effort Noise	Effort	Clan
	High Effort noise	Outcome or Effort	Clan

The main difference between Tables 4.4 and 4.5 is that, in contrast to Govindarajan and Fisher (1990), our results suggest clan control as optimal if task noise is high. In other cases, our model suggests either outcome or effort control as optimal depending strictly on the relative noisiness of the corresponding signals, while the

Govindarajan and Fisher study strictly favor effort control when both types of signals are noisy when task noise is low. We argue that this difference comes from the vague distinction between effort noise and task noise in the Govindarajan and Fisher study.

4.6 CONCLUSION

We develop an analytical model with factors including outcome, effort, and task noise, and socialization cost based on the setting of moral hazard and decision delegation under post-contract information asymmetry, and we apply it to precisely identify which management control mechanism will be optimal under given circumstances, and why. In doing so, we extend the agency theory on management control to incorporate the notion of other-regarding preference and clan control, and we sharpen and extend the literature on organizational control. We show that the optimal management control mechanism depends on the relative magnitude of task uncertainty, outcome noise, effort noise, and socialization cost.

This paper makes several important contributions to the literature. First, we recast the problem of optimal organizational control into the framework of an agency model, enabling us to more precisely define various concepts of information and uncertainty that are relevant to the problem. We distinguish outcome measurement noise from production noises (effort and task)— a distinction that is often not clarified in the organization control literature. In addition, we provide a precise definition of task programmability to distinguish it from other types of measurability that are exogenously risen. By developing a rigorous model, we provide a tool for analyzing interesting issues in organizational control. Second, we broaden the scope of traditional agency theory by introducing a new form of organizational control (clan control), an aligned agent, who partially shares the goals of the principal, and the concept of a socialization cost. While the notion of an aligned agent/steward is not widely accepted in accounting and economic literature, in many situations, managers, researchers, government and not-for-profit organization officials, and others may be given a fair amount discretion in making decisions (recall the degree of alignment is not necessarily one). In situations where optimal decisions are not easily determined ex ante, and therefore contracts are incomplete, promoting goal alignment by socialization could be a viable alternative control mechanism. Third, by applying agency theory to organization theory, we expand the findings of Ouchi (1979), Eisenhardt (1985), Govindarajan and Fisher (1990), and others. We reiterate the contribution made by organization theory in expanding our horizon in thinking about control mechanisms. The economics paradigm is often too explicit and narrow because of our desire to maintain analytically rigorous. But in so doing, it is possible that economic models have ignored certain fundamental human traits.

CHAPTER 5

CONCLUSION

The first and second study contribute to the literature by focusing on the value of pre-contract information on ability in a setting with both moral hazard and adverse selection. In our model, the principal's optimal contract is calibrated after the principal observes a signal(s) of the agent's ability but before a menu of contract is offered and accepted. The setting with pre-contract information enables us to examine the role and the value of information obtained by the principal about a particular agent prior to designing a compensation scheme. In practice, such information is routinely gathered during the search for an agent as well as in many other analogous situations, underscoring the importance of this type of information. Finally, although we speak throughout the paper in terms of a standard principal-agent relationship, and we equate *type* with *ability*, our results can be readily adapted to other settings that entail both adverse selection and moral hazard. For example, instead of a principal-owner negotiating a salary and a contingent bonus with a prospective agent-manager who has private knowledge of his own ability, consider an investor negotiating with an entrepreneur/manager over the terms of acquiring his startup company, of which only the manager knows the true quality. Our model can accommodate this scenario by reinterpreting the salary as a fixed component of the purchase agreement and the bonus as a contingent component of the agreement, where the manager commits to manage the company for a specified period of time after the acquisition, and where the contingent payment will depend on the firm's performance during this post-acquisition period. Our model has interesting empirical predictions in this case as well, including the type of information the principal will be willing to bear a cost to obtain (or the agent will be willing to bear a cost to signal), how the terms of such an agreement should depend on the features of the firm and the information environment, and in which types of industries (and for which types of firms) such agreements should be more relevant and likely to be adopted. For the second study, we contribute to the literature by examining the relative weights in an adverse selection setting. Our analysis should provide guidance as to how to evaluate the value of information about ability, which will be useful for the principal in deciding how much to invest in acquiring such information and how to use the acquired information. In addition, a principal possessing multiple pieces of information would want to aggregate the information, and our analysis can be extended to handle this case. While we have treated information as a scalar in the first study for simplicity, we relax this assumption in the second study and address the question of how to assign relative weighs to different pieces of information, such as the agent's educational background, professional qualifications, and past performance in the firm he worked, or in the current firm.

For our third study, we develop an analytical model with factors including outcome, behavior, and task noise, and socialization cost based on the setting of moral hazard and decision delegation under post-contract information asymmetry, and we apply it to precisely identify which control strategies will be more (less) likely adopted given the changes in the agency relationship. We expect to make several contributions to the literature. First, we render the problem of optimal organizational control in the framework of an agency model, which allows us to more precisely define the various concepts of information and uncertainty that are relevant to the problem. We distinguish outcome measurement noise from production noise, one is oftentimes confused with task noise in the organization control literature, and we show how they enter similarly into the optimal strategy problem. We also precisely define task noise, and distinguish it from other types of uncertainty with which it is sometimes conflated in the literature. Second, we illuminate the fundamental roles of the outcome and behavior signals in agency models in determining the optimal control strategy. Third, we broaden the scope of traditional agency theory by introducing a new form of organizational control (clan control); a congruent agent, who partially shares the goals of the principal; and the concept of a socialization cost. Fourth, we introduce task-specific post-contract information asymmetry and construct tractable analytical models, which is new in the agency literature.

BIBLIOGRAPHY

Adams. R., H. Almeida, and D. Ferreira. 2005. Powerful CEOs and their impact on corporate performance. *Review of Financial Studies*, 18(4), 1403-32.

Aggarwal, R. K., and A. A. Samwick. 1999. The other side of the trade-off: the impact of risk on executive compensation. *Journal of Political Economy*, 107(1), 65-105.

Agrawal, A., and C. Knoeber. 2001. Do some outside directors play a political role? *Journal of Law and Economics*, 44, 179-98.

Amershi, A. H., R. D. Banker, and S. M. Datar. 1990. Economic sufficiency and statistical sufficiency in the aggregation of accounting signals. *The Accounting Review*, 65(1), 113-130.

Andreoni, J. 1989. Giving with impure altruism: Applications to charity and Ricardian equivalence. *Journal of political Economy*, *97*(6), 1447-1458.

Andreoni, J., & Miller, J. 2002. Giving according to GARP: An experimental test of the consistency of preferences for altruism. *Econometrica*, 70(2), 737-753.

Antle, R., and J. Fellingham. 1995. Resource rationing and organizational slack in a two-period Model. *Journal of Accounting Research*, 28(1), 1-24.

Bagnoli, M., and T. Bergstrom. 2005. Log-concave probability and its applications. *Economic Theory*, 26 (2), 445-69.

Banker, R. and S. Datar. 1989. Sensitivity, precision, and linear aggregation of signals for performance evaluation. *Journal of Accounting Research*, 27, 21-39.

Banker, R., Darrough, M., Li, S., & Threinen, L. 2019. The Value of Precontract Information About an Agent's Ability in the Presence of Moral Hazard and Adverse Selection. *Journal of Accounting Research*, 57(5), 1201-1245.

Bernardo, A. E., H. Cai, and J. Luo. 2001. Capital budgeting and compensation with asymmetric information and moral hazard. *Journal of Financial Economics*, 61, 311-344.

Besley, T. and M. Ghatak. 2005. Competition and incentives with motivated agents. *The American Economic Review*, 95(3), 616-636.

Bethwaite, J., and Tompkinson, P. 1996. The ultimatum game and non-selfish utility functions. *Journal of Economic Psychology*, 17(2), 259-271.

Bewley, T. (2009). Why wages don't fall during a recession. Harvard university press.

Biglaiser, G., and C. Mezzetti. 1993. Principals competing for an agent in the presence of adverse selection and moral hazard. *Journal of Economic Theory*, 61, 302-330.

Bizjak, J., M. Lemmon, and L. Naveen. 2008. Does the use of peer groups contribute to higher pay and less efficient compensation? *Journal of Financial Economics*, 90, 152-168.

Bolton, P., and M. Dewatripont. 2005. Contract Theory. Cambridge, MA: MIT Press.

Bushman, R., and R. Indjejikian. 1993. Accounting income, stock price, and managerial compensation. *Journal of Accounting and Economics*, 16 (1-3), 3-23.

Bushman, R., R. Indjejikian, and A. Smith, 1996. CEO compensation: the role of individual performance evaluation. *Journal of Accounting and Economics*, 21, 161-193.

Charness, G. and M. Rabin. 2002. Understanding social preferences with simple tests. *Quarterly Journal of Economics*, 117, 817–869.

Christensen, P. O., F. Şabac, and J. Tian. 2010. Ranking performance measures in multi-task agencies. *The Accounting Review*, 85(5), 1545-1575.

Coase, R. 1960. The problem of social cost. In Classic papers in natural resource economics (pp. 87-137). Palgrave Macmillan, London.

Conyon, M., and K. J. Murphy. 2000. The prince and the pauper? CEO pay in the United States and United Kingdom. *The Economic Journal*, 110 (467), 640- 671.

Core, J. and W. Guay. 1999. The use of equity grants to manage optimal equity incentive levels. *Journal of Accounting and Economics*, 28(2), 151-184.

Core, J., W. Guay, and R. Verrecchia. 2003. Price versus non-price performance measures in optimal CEO compensation contracts. *The Accounting Review*, 78(4), 957–981.

Darrough, M., and N. Melumad. 1995. Divisional versus company-wide focus: The trade-off between allocation of managerial attention and screening of talent. *Journal of Accounting Research*, 33 (Supplement), 65-94.

Datar, S., S. Kulp, and R. Lambert. 2001. Balancing performance measures. *Journal of Accounting Research*, 39 (1), 75-92.

Dufwenberg, M. and G. Kirchsteiger. 2004. A theory of sequential reciprocity. Games and *Economic Behavior*, 47, 268-298.

Dutta, S. 2008. Managerial expertise, private information, and pay-performance sensitivity. *Management Science*, 54 (3), 429-442.

Eisenhardt, K. M. 1985. Control: organizational and economic approaches. *Management Science*, 31(2), 134-149.

Eisenhardt, K. M. 1988. Agency- and institutional-theory explanations: the case of retail sales compensation. *The Academy of Management Journal*, 31(3), 488-511.

Eisenhardt, K. M. 1989. Agency theory - An assessment and review. *Academy of Management Review*, 14 (1), 57-74.

Erlei, M. 2008. Heterogeneous social preferences. *Journal of Economic Behavior & Organization*, 65, 436-457.

Fama, E. 1980. Agency problems and the theory of the firm. *Journal of Political Economy*, 88, 288-307.

Fehr, E. and A. Falk. 2002. Psychological foundations of incentives. *European Economic Review*, 46, 687-724.

Fehr, E. and K. M. Schmidt. 2006. The economics of fairness, reciprocity and altruism – experimental evidence and new theories. *Handbook on the Economics of Giving, Reciprocity and Altruism*, 1, 615-691.

Feltham, G., and J. Xie. 1994. Performance measure congruity and diversity in multi-task principal/agent relations. *The Accounting Review*, 69 (3), 429-453.

Gabaix, X., and A. Landier. 2008. Why has CEO pay increased so much? *Quarterly Journal of Economics*, 123(1). 49-100.

Garen, J. 1994. Executive compensation and principal-agent theory. *Journal of Political Economy*, 102, 1175-1199.

Gintis, H. 2000. Beyond Homo economicus: evidence from experimental economics. *Ecological economics*, 35(3), 311-322.

Gintis, H., Bowles, S., Boyd, R., and Fehr, E. 2003. Explaining altruistic behavior in humans. *Evolution and human Behavior*, 24(3), 153-172.

Glaeser, E., and D. Mare. 2001. Cities and skills. *Journal of Labor Economics*, 19(2), 316-342.

Gould, E. 2005. Inequality and expertise. *Labor Economics*, 12(2), 169-189.

Govindarajan, V. and J. Fisher. 1990. Strategy, control systems, and resource sharing: effects on business-unit performance. *The Academy of Management Journal*, 33(2), 259-285.

Güth, W., Schmittberger, R., & Schwarze, B. 1982. An experimental analysis of ultimatum bargaining. *Journal of Economic Behavior & Organization*, 3(4), 367-388.

Hambrick, D., and P. Mason. 1984. Upper echelons: The organization as a reflection of its top managers. *Academy of Management Review*, 9, 193-206.

Hamilton, W. 1964. The genetical evolution of social behaviour. II. *Journal of Theoretical Biology*, 7(1), 17-52.

Harris, M., and A. Raviv. 1978. Some results on incentive contracts with applications to education and employment, health insurance, and law enforcement. *American Economic Review*, 68 (1), 20-30.

Healy, P. 1985. The effect of bonus schemes on accounting decisions. *Journal of Accounting and Economics*, 7, 85-107.

Henderson, A. D., and J. W. Fredrickson. 1996. Information-processing demands as a determinant of CEO compensation. *The Academy of Management Journal*, 39(3), 575-606.

Henrich, J., Boyd, R., Bowles, S., Camerer, C., Fehr, E., Gintis, H., and McElreath, R. 2001. Cooperation, reciprocity and punishment in fifteen small-scale societies. *American Economic Review*, 91(2), 73-78.

Holmström, B. 1979. Moral hazard and observability. *Bell Journal of Economics*, 10, 74-91.

Holmström, B. 1982. Moral hazard in teams. *Bell Journal of Economics*, 13(2), 324-340.

Holmström, B., and P. Milgrom. 1987. Aggregation and linearity in the provision of intertemporal incentives. *Econometrica*, 55(March), 303-328.

Huddart, S. and P. J., Liang. 2003. Accounting in partnerships. *American Economic Review*, 93(2), 410-414.

Inderst, R. 2001. Incentive schemes as a signaling device. *Journal of Economic Behavior & Organization*, 44, 455-465.

Ittner, C., D. Larcker, and M. Rajan. 1997. The choice of performance measures in annual bonus contracts. *The Accounting Review*, 72, 231-255.

Jensen, M. and J. Zimmerman. 1985. Management compensation and the managerial labor market. *Journal of Accounting and Economics*, 7, 3-9.

Jensen, M., and W. Meckling. 1976. Theory of the firm: managerial behavior, agency costs and ownership structure. *Journal of Financial Economics*, 3, 305-360.

Jin, L. 2002. CEO compensation, diversification, and incentives. *Journal of Financial Economics*, 66, 29- 63.

Kim, S. K, and Y. S. Suh. 1991. Ranking of accounting information systems for management control. *Journal of Accounting Research*, 29 (2), 386-396.

Kirsch, L. 1996. The management of complex tasks in organizations: controlling the systems development process. *Organization Science*, 7(1), 1-21.

Krishna, V. 2002. Auction theory. Academic Press.

Laffont, J., and J. Tirole. 1993. A theory of incentives in procurement and regulation. MIT Press, Cambridge, MA.

Lambert, R., and D. Larcker. 1987. An analysis of the use of accounting and market measures of performance in executive compensation contract. *Journal of Accounting Research*, 25 (Supplement), 85-129.

Lazear, E. 2000. Performance pay and productivity. *American Economic Review*, 90(5), 1346-1361.

Lazear, E. 1986. Salaries and piece rates. *Journal of Business*, 59(3), 405-431.

Levine, D. 1998. Modeling altruism and spitefulness in experiments. *Review of Economic Dynamics*, 1, 593-622.

Malmendier, U., and G. Tate. 2009. Superstar CEOs. *Quarterly Journal of Economics*, 124(4), 1593-1638.

Myerson, R. 1979. Incentive compatibility and the bargaining problem. *Econometrica*, 47 (1), 61-73.

Myerson, R. 1981. Optimal auction design. *Mathematics of Operations Research*, 6 (1), 58-73.

Ouchi, W. 1979. A conceptual framework for the design of organizational control mechanisms. *Management Science*, 25(9), 833-848.

Ouchi, W. 1980. Markets, bureaucracies, and clans. *Administrative Science Quarterly*, 25(1), 129-141.

Ouchi, W. G. 1981. Theory Z. Addison-Wesley Reading, MA.

Oyer, P. and S. Schaefer. 2005. Why do some firms give stock options to all employees?: An empirical examination of alternative theories, *Journal of Financial Economics*, 76, 99-133.

Rabin, M. 1993. Incorporating fairness into game theory and economics. *American Economic Review*, 83(5),1281-1302.

Rajgopal, S. T. Shevlin, and V. Zamora. 2006. CEOs' outside employment opportunities and the lack of relative performance evaluation in compensation contracts. *Journal of Finance*, 61 (4), 1813-1844.

Rose, N., and A. Shepard. 1997. Firm diversification and CEO compensation: managerial ability or executive entrenchment? *The RAND Journal of Economics*, 28 (3), 489-514

Rotemberg, J. 2008. Minimally acceptable altruism and the ultimatum game. *Journal of Economic Behavior & Organization*, 66, 457-476.

Roth, A, Malouf, M., & Murnighan, J. 1981. Sociological versus strategic factors in bargaining. *Journal of Economic Behavior & Organization*, 2(2), 153-177.

Rothschild, M., and J. Stiglitz. 1976. Equilibrium in competitive insurance markets: an essay on the economics of imperfect information. *Quarterly Journal of Economics*, 90 (4), 629-649.

Salanie, B. 2005. The Economics of Contracts. Cambridge, MA: MIT Press.

Salop, J., and S. Salop. 1976. Self-selection and turnover in the labor market. *Quarterly Journal of Economics*, 90 (4), 619-627.

Sloan, R. 1993. Accounting earnings and top executive compensation. *Journal of Accounting and Economics*, 16(1-3), 55-100.

Sloan, R. 1993. Accounting earnings and top executive compensation. *Journal of Accounting and Economics*, 16(1-3), 55-100.

Spence, M. 1973. Job market signaling. *Quarterly Journal of Economics*, 87, 355-374.

Terviö, M. 2008. The difference that CEOs make: An assignment model approach. *American Economic Review*, 98(3), 642-668.

Thompson, J. 1967. Organizations in Action, McGraw-Hill, New York.

Trivers, R. 1971. The evolution of reciprocal altruism. *The Quarterly Review of Biology*, 46(1), 35-57.

Tushman, M. and L. Romanelli. 1985. Organizational evolution: A metamorphosis model of convergence and reorientation. *Research in Organizational Behavior*, 7, 171-223.

Wilson, C. 1977. A model of insurance markets with incomplete information. *Journal of Economic Theory*, 16 (2), 167-207.

Yermack, D. 1995. Do corporations award CEO stock options effectively? *Journal of Financial Economics,* 39, 237-269.

APPENDIX A

PROOFS

PROOF OF LEMMA 2.1 (Solution to the Principal's Problem)

Given α and β, the agent's problem is effectively to choose an effort level to maximize his certainty equivalent

$$CE\left(w\big(a, y(e,a)\big)\right) = \alpha + \beta(\lambda a + e) - \frac{R\beta^2\sigma_y^2}{2} - \frac{ce^2}{2}, a \in \{H, L\}. \qquad (A.1)$$

The solution to the effort problem for a particular a is then

$$e(a) = \frac{\beta_a\gamma}{c}, a \in \{H, L\}. \qquad (A.2)$$

Note that this result holds only for $\beta \geq 0$; otherwise, the solution is simply $e(a) = 0$, $a \in \{H,L\}$. The individual rationality (IR) constraint for the L type agent binds, so that

$$\alpha_L + \beta_L\big(\lambda L + e(L)\big) - \frac{R\beta_L^2\sigma_y^2}{2} - \frac{ce(L)^2}{2} = r_0. \qquad (A.3)$$

Given this, we can express α_L as

$$\alpha_L = r_0 - \beta_L\big(\lambda L + e(L)\big) + \frac{R\beta_L^2\sigma_y^2}{2} + \frac{ce(L)^2}{2}. \qquad (A.4)$$

The truth-telling (TT) constraint of the H type agent also binds, so that

$$\alpha_H + \beta_H\big(\lambda H + e(H)\big) - \frac{R\beta_H^2\sigma_y^2}{2} - \frac{ce(H)^2}{2}$$

$$= \alpha_L + \beta_L\big(\lambda H + e(L)\big) - \frac{R\beta_L^2\sigma_y^2}{2} - \frac{ce(L)^2}{2}. \qquad (A.5)$$

Given this, we can express α_H as

$$\alpha_H = \alpha_L + \beta_L\big(\lambda H + \gamma e(L)\big) - \frac{R\beta_L^2\sigma_y^2}{2} - \frac{ce(L)^2}{2}$$

$$-\beta_H\big(\lambda H + \gamma e(H)\big) + \frac{R\beta_H^2\sigma_y^2}{2} + \frac{ce(H)^2}{2}. \qquad (A.6)$$

The principal's objective function is

$$\max_{\{\beta_H, \beta_L, e(a)\}} x\left[\lambda H + e(H) - \left(\alpha_H + \beta_H\big(\lambda H + \gamma e(H)\big)\right)\right]$$

$$+ (1 - x)\left[\lambda L + e(L) - \left(\alpha_L + \beta_L\big(\lambda L + e(L)\big)\right)\right]. \qquad (A.7)$$

Combining equations (A.5), (A.6), and (A.7) allows us to solve for the optimal values of β_H and β_L. Note that it is here that the non-negativity constraint on effort (in particular, on β_L) gives rise to the threshold. Plugging those values back in to equations (A.3) and (A.4) immediately gives us the optimal values of α_H and α_L.

Further, plugging these optimal values in to equations (A.1) and (A.2) and taking the expectation over ε gives us the expected compensation for each type; and subtracting this expected compensation from expected production gives the expressions for expected profit by type. Finally, taking the expected value of profit over the agent types gives equation (2.16).

PROOF OF LEMMA 2.3 (Total Profit and Information about Ability)

Taking the derivative of equation (2.16) with respect to z gives

$$\frac{\partial(E_a[\pi(a)])}{\partial z} = \left[\lambda(H - L) + \frac{A^2}{2c\big(1 + cR\sigma_y^2\big)}\right]\frac{\partial u}{\partial z}$$

$$+ \left(2c\big(1 + cR\sigma_y^2\big)\right)^{-1}\left[A^2\frac{u}{1-u}\left(2 + \frac{u}{1-u}\right) - 2A - 1\right]\frac{\partial u}{\partial z}, \qquad (A.8)$$

where the second bracketed term appears only if $x < \bar{x}$. Clearly the first bracketed term is positive, and $\frac{\partial u}{\partial z}$ is positive as well due to Assumption 2.1, so we shall prove that the sum of both terms is also positive. Proceeding,

$$\left[\lambda(H-L)+\frac{1}{2c\left(1+cR\sigma_y^2\right)}\right]\frac{\partial u}{\partial z}$$

$$+\left(2c(1+cR\sigma^2)\right)^{-1}\left[A^2\frac{u}{1-u}\left(2+\frac{u}{1-u}\right)-2A-1\right]\frac{\partial u}{\partial z}$$

$$=\left[\frac{A}{c}+\left(2c\left(1+cR\sigma_y^2\right)\right)^{-1}\left(A^2\frac{u}{1-u}\left(2+\frac{u}{1-u}\right)-2A\right)\right]\frac{\partial u}{\partial z}$$

$$=\frac{A}{c}\left[1+\frac{A\frac{u}{1-u}\left(2+\frac{u}{1-u}\right)}{2\left(1+cR\sigma_y^2\right)}-\frac{1}{1+cR\sigma_y^2}\right]\frac{\partial u}{\partial z}$$

$$=\frac{A}{c}\left[\frac{cR\sigma^2}{1+cR\sigma_y^2}+\frac{A\frac{u}{1-u}\left(2+\frac{u}{1-u}\right)}{2\left(1+cR\sigma_y^2\right)}\right]\frac{\partial u}{\partial z}\geq 0.$$

PROOF OF LEMMA 2.4 (Value of Information about Ability)

Below, we give a full derivation of equation (2.20). The proof of equation (2.21) is similar.

We have defined the value of information as the difference between *ex ante* expected profit when the information will be observed on one hand, and expected profit when the information will not be observed on the other. When $v \leq \bar{v}$, Lemma 2.1 implies that upon observing information $z \leq \bar{z}$, the expected profit will be given by the first expression on the righthand side of equation (2.16) (using the principal's posterior belief u); whereas upon observing information $z > \bar{z}$, the expected profit will be given by the second expression in the same equation (where in both cases, the principal's posterior belief u is used in place of x). The *ex ante* value of the information can thus be expressed, taking the integral over all possible realized values of the information, as

$$E[V(z \mid v \leq \bar{v})]=\int_{-\infty}^{\bar{z}}\left(\lambda(uH+(1-u)L)+\frac{u}{2c\left(cR\sigma_y^2+1\right)}-r_0\right.$$

$$-u\frac{2A\left(1-A\frac{u}{1-u}\right)}{2c\left(cR\sigma_y^2+1\right)}+(1-u)\frac{\left(1-\left(A\frac{u}{1-u}\right)^2\right)}{2c\left(cR\sigma_y^2+1\right)}\Bigg)g(z)dz$$

$$+\int_{\bar{z}}^{\infty}\left(\lambda(uH+(1-u)L)+\frac{u}{2c\left(cR\sigma_y^2+1\right)}-r_0\right)g(z)dz$$

$$-\Bigg(\lambda(vH+(1-v)L)+\frac{v}{2c\left(cR\sigma_y^2+1\right)}-r_0$$

$$-v\frac{2A\left(1-A\frac{v}{1-v}\right)}{2c\left(cR\sigma_y^2+1\right)}+(1-v)\frac{\left(1-\left(A\frac{v}{1-v}\right)^2\right)}{2c\left(cR\sigma_y^2+1\right)}\Bigg).$$

Several of the terms in this expression cancel immediately, essentially because the fact of observing the information does not affect the probability distribution of agent types in the population. Simplifying in this way gives

$E[V(z \mid v \leq \bar{v})]$

$$=\int_{-\infty}^{\bar{z}}\left(-u\frac{2A\left(1-A\frac{u}{1-u}\right)}{2c\left(cR\sigma_y^2+1\right)}+(1-u)\frac{\left(1-\left(A\frac{u}{1-u}\right)^2\right)}{2c\left(cR\sigma_y^2+1\right)}\right)g(z)dz$$

$$-\left(-v\frac{2A\left(1-A\frac{v}{1-v}\right)}{2c\left(cR\sigma_y^2+1\right)}+(1-v)\frac{\left(1-\left(A\frac{v}{1-v}\right)^2\right)}{2c\left(cR\sigma_y^2+1\right)}\right)$$

$$=\left(2c\left(cR\sigma_y^2+1\right)\right)^{-1}$$

$$\cdot\Bigg[2A\left(v-\int_{-\infty}^{\bar{z}}ug(z)dz\right)$$

$$+\left(\int_{-\infty}^{\bar{z}}(1-u)g(z)dz-(1-v)\right)+A^2\left(\int_{-\infty}^{\bar{z}}\frac{u^2}{1-u}g(z)dz-\frac{v^2}{1-v}\right)\Bigg].$$

Next, note that $g(z) = vg_H(z) + (1-v)g_L(z)$ which, together with equation (2.3), implies that $u = \frac{g_H(z)}{g(z)}v$ and $1-u = \frac{g_L(z)}{g(z)}(1-v)$. Applying this result, along with equation (2.4), to the preceding equation gives

$$E[V(z \mid v \leq \overline{v})]$$

$$= \left(2c\left(cR\sigma_y^2 + 1\right)\right)^{-1}$$

$$\cdot\left[2A\left(v - \int_{-\infty}^{\overline{z}} ug(z)dz\right)\right.$$

$$\left. + \left(\int_{-\infty}^{\overline{z}} (1-u)g(z)dz - (1-v)\right) + A^2\left(\int_{-\infty}^{\overline{z}} \frac{u^2}{1-u}g(z)dz - \frac{v^2}{1-v}\right)\right]$$

$$= \left(2c\left(cR\sigma_y^2 + 1\right)\right)^{-1}$$

$$\cdot\left[2Av\left(1 - \int_{-\infty}^{\overline{z}} g_H(z)dz\right) + (1-v)\left(\int_{-\infty}^{\overline{z}} g_L(z)dz - 1\right)\right.$$

$$\left. + A^2\frac{v^2}{1-v}\left(\int_{-\infty}^{\overline{z}} \frac{g_H(z)^2}{g_L(z)}dz - 1\right)\right]$$

$$= \left(2c\left(cR\sigma_y^2 + 1\right)\right)^{-1}$$

$$\cdot\left[2Av\left(1 - G_H(z)\right)\right.$$

$$\left. - (1-v)\left(1 - G_L(z)\right) + A^2\frac{v^2}{1-v}\left(\int_{-\infty}^{\overline{z}} \frac{g_H(z)^2}{g_L(z)}dz - 1\right)\right].$$

which is equivalent to the desired expression.

PROOF OF PROPOSITION 2.2 (Value of Information about Ability and the Severity of Adverse Selection)

We prove here that the value of the information is quasi-concave in λ when $v \leq \bar{v}$. The proof for the alternate case is similar, as are the proofs for the equivalent proposition regarding $(H-L)$. Taking the partial derivative of equation (2.20) with respect to λ gives

$$\frac{\partial\left[E[V(z|\, v \leq \bar{v})]\right]}{\partial\lambda} = \left(2c\left(cR\sigma_y^2+1\right)\right)^{-1}$$

$$\cdot\left[2A\frac{\partial A}{\partial\lambda}\cdot\frac{v^2}{1-v}\left(\int_{-\infty}^{\bar{z}}\frac{g_H(z)^2}{g_L(z)}dz-1\right)+2\frac{\partial A}{\partial\lambda}\cdot v\left(1-G_H(\bar{z})\right)\right.$$

$$\left.+\left(A^2\cdot\frac{v^2}{1-v}\frac{g_H(\bar{z})^2}{g_L(\bar{z})}-2Av\cdot g_H(\bar{z})+(1-v)\cdot g_L(\bar{z})\right)\frac{\partial\bar{z}}{\partial\lambda}\right].$$

Noting that $\frac{g_H(\bar{z})}{g_L(\bar{z})}=\frac{A}{B\frac{v}{1-v}}$ by definition, the term in parentheses multiplying $\partial\bar{z}/\partial\lambda$ is equal to zero, an implication of the envelope theorem. Thus,

$$\frac{\partial\left[E[V(z|\, v \leq \bar{v})]\right]}{\partial\lambda}$$

$$=\left(2c\left(cR\sigma_y^2+1\right)\right)^{-1}\cdot\left[2A\frac{\partial A}{\partial\lambda}\cdot\frac{v^2}{1-v}\left(\int_{-\infty}^{\bar{z}}\frac{g_H(z)^2}{g_L(z)}dz-1\right)+2\frac{\partial A}{\partial\lambda}\cdot v\left(1-G_H(\bar{z})\right)\right]$$

$$=C_1\left[A\frac{v}{1-v}\left(\int_{-\infty}^{\bar{z}}\frac{g_H(z)^2}{g_L(z)}dz-1\right)+\left(1-G_H(\bar{z})\right)\right], \qquad (A.9)$$

where C_1 is a positive constant that does not depend on λ. Thus, critical points occur when $\left[A\frac{v}{1-v}\left(\int_{-\infty}^{\bar{z}}\frac{g_H(z)^2}{g_L(z)}dz-1\right)+\left(1-G_H(\bar{z})\right)\right]=0$. Further,

$$\frac{\partial^2\left[E[V(z|\, v \leq \bar{v})]\right]}{\partial\lambda^2}=C_2\left[\int_{-\infty}^{\bar{z}}\frac{g_H(z)^2}{g_L(z)}dz-1\right],$$

where C_2 is a positive constant. Thus, the curvature of the value function is determined by the sign of $\int_{-\infty}^{\bar{z}}\frac{g_H(z)^2}{g_L(z)}dz-1$. However, equation (A.9) stipulates that, at all critical points,

$$\int_{-\infty}^{\bar{z}} \frac{g_H(z)^2}{g_L(z)} dz - 1 = -\frac{1-v}{Av}\left(1 - G_H(\bar{z})\right) < 0,$$

so that the second derivative is strictly negative. Thus, all critical points are local maxima, from which it follows that there is only a single critical point, and it is a maximum.

PROOF OF LEMMA 2.5 (Sensitivity of Expected Profit to Information about Ability)

Taking the derivative of equation (2.26) with respect to $\lambda(H-L)$ gives

$$\frac{\partial^2 E[\pi_H]}{\partial LR(z)\, \partial[\lambda(H-L)]} = \frac{2\dfrac{Av}{1-v}}{cR\sigma_y^2 + 1} > 0.$$

Because the expression in equation (2.26) is also greater than zero, this result shows that the magnitude of the effect of the likelihood ratio on π_H is increasing in $\lambda(H-L)$—that is, in the severity of adverse selection. Similarly for equation (2.27),

$$\frac{\partial^2 E[\pi_L]}{\partial LR(z)\, \partial[\lambda(H-L)]} = -\frac{2A\left(\dfrac{v}{1-v}\right)^2 LR(z)}{cR\sigma_y^2 + 1} < 0, \qquad (A.10)$$

which, like the expression in equation (2.27), is less than zero. This shows that the magnitude of the effect of the likelihood ratio on π_L is also increasing (i.e., becomes more negative) in $\lambda(H-L)$.

PROOF OF LEMMA 2.6 (Location of Peak Information Value)

Let ζ be an arbitrary parameter not affecting the information distribution. The maximum value of information is attained when $\left[\int_{-\infty}^{\bar{z}} \frac{g_H(z)^2}{g_L(z)} dz - 1 + \frac{1-v}{Av}\left(1 - G_H(\bar{z})\right)\right] = 0$ (see proof of Proposition 2.2). Then the effect of a change in ζ on the value of $[\lambda(H-L)]_{\max}$ follows

$$\frac{g_H(\overline{z})^2}{g_L(\overline{z})}\frac{\partial \overline{z}}{\partial \zeta} + \left(\frac{\partial \left(\frac{1-v}{Av}\right)}{\partial \zeta} + \frac{\partial \left(\frac{1-v}{Av}\right)}{\partial \big(\lambda(H-L)\big)} \frac{\partial \big(\lambda(H-L)\big)_{\max}}{\partial \zeta} \right) \big(1 - G_H(\overline{z})\big)$$

$$- \frac{1-v}{Av} g_H(\overline{z}) \frac{\partial \overline{z}}{\partial \zeta} = 0$$

The first and last terms in this expression cancel one another, giving

$$\frac{\partial \big(\lambda(H-L)\big)_{\max}}{\partial \zeta} = - \frac{\dfrac{\partial \left(\frac{1-v}{Av}\right)}{\partial \zeta}}{\dfrac{\partial \left(\frac{1-v}{Av}\right)}{\partial \big(\lambda(H-L)\big)}} ,$$

Implying

$$\frac{\partial \log \big(\lambda(H-L)\big)_{\max}}{\partial \log(\zeta)} = - \frac{\dfrac{\partial \log \left(\frac{1-v}{Av}\right)}{\partial \log(\zeta)}}{\dfrac{\partial \log \left(\frac{1-v}{Av}\right)}{\partial \log \big(\lambda(H-L)\big)}} .$$

Likewise, the critical value $\overline{\lambda(H-L)}$ occurs when $\frac{1-v}{Av} = 1$, so that the effect of a change in ζ on $\overline{\lambda(H-L)}$ follows

$$\frac{\partial \overline{\lambda(H-L)}}{\partial \zeta} = - \frac{\dfrac{\partial \left(\frac{1-v}{Av}\right)}{\partial \zeta}}{\dfrac{\partial \left(\frac{1-v}{Av}\right)}{\partial \big(\lambda(H-L)\big)}} ,$$

implying

$$\frac{\partial \log \left(\overline{\lambda(H-L)}\right)}{\partial \log(\zeta)} = - \frac{\dfrac{\partial \log \left(\frac{1-v}{Av}\right)}{\partial \log(\zeta)}}{\dfrac{\partial \log \left(\frac{1-v}{Av}\right)}{\partial \log \big(\lambda(H-L)\big)}} .$$

The right-hand side expressions of these two equations are identical, but they are evaluated at different values (using $[\lambda(H-L)]_{\max}$ and $\overline{\lambda(H-L)}$, respectively). If they are equal irrespective of the value at which they are evaluated, then a log change in ζ will induce identical log changes in $[\lambda(H-L)]_{\max}$ and in $\overline{\lambda(H-L)}$, respectively, giving the result. And because all parameters enter $\frac{1-v}{Av}$ multiplicatively, this is indeed true.

PROOF OF PROPOSITION 2.4 (Precision and Sensitivity)

Below, we give a full derivation based on equation (2.29). The proof using equation (2.30) is similar.

To prove the precision result, we will use the facts that for a normal distribution with mean θ_a, $\frac{\partial\phi_a(z)}{\partial\left(\frac{1}{\sigma^2}\right)} = -\frac{\sigma^4}{2}\frac{\partial^2\phi_a(z)}{\partial z^2}$ and $\frac{\partial\phi_a(z)}{\partial z} = -\frac{z-\theta a}{\sigma^2}\phi_a(z)$. We begin by plugging the pdf of the normal distribution in to equation (2.20) and taking the partial derivative with respect to $\frac{1}{\sigma_z^2}$, giving[57]

$$\partial(E[V(z \mid v \leq \overline{v})])$$

$$= \left(2c\left(cR\sigma_y^2+1\right)\right)^{-1}\left[A^2\frac{v^2}{1-v}\int_{-\infty}^{\bar{z}}\left(2\frac{\partial\left(\phi_H(z)\right)}{\partial\left(\frac{1}{\sigma_z^2}\right)}\frac{\phi_H(z)}{\phi_L(z)} - \frac{\partial\left(\phi_L(z)\right)}{\partial\left(\frac{1}{\sigma_z^2}\right)}\left(\frac{\phi_H(z)}{\phi_L(z)}\right)^2\right)dz\right.$$

$$\left. -2Av\int_{-\infty}^{\bar{z}}\frac{\partial\left(\phi_H(z)\right)}{\partial\left(\frac{1}{\sigma_z^2}\right)}dz + (1-v)\int_{-\infty}^{\bar{z}}\frac{\partial\left(\phi_L(z)\right)}{\partial\left(\frac{1}{\sigma_z^2}\right)}dz\right]. \qquad (A.11)$$

---

[57] Note that, while the value of the threshold $\bar{z}$ is affected by the information's precision, there is no resulting effect on the information's value, as in the proof of Proposition 2.2. The same is true in the proof of the sensitivity result below.

The last two terms in the bracketed expression can be evaluated directly by using the facts noted above. The others can be evaluated using repeated integration by parts. We illustrate using the first term in parentheses. We have

$$\int_{-\infty}^{\bar{z}} \frac{\partial(\phi_H(z))}{\partial\left(\frac{1}{\sigma_z^2}\right)} \frac{\phi_H(z)}{\phi_L(z)} dz = \int_{-\infty}^{\bar{z}} -\frac{\sigma_z^4}{2} \frac{\partial^2(\phi_H(z))}{\partial z^2} e^{\left[\frac{\theta(H-L)}{\sigma_z^2}\left(z-\frac{\theta(H+L)}{2}\right)\right]} dz$$

$$= -\frac{\sigma_z^4}{2}\left(\frac{\partial(\phi_H(\bar{z}))}{\partial z} e^{\left[\frac{\theta(H-L)}{\sigma_z^2}\left(\bar{z}-\frac{\theta(H+L)}{2}\right)\right]} -\right.$$

$$\left.-\int_{-\infty}^{\bar{z}} \frac{\partial(\phi_H(z))}{\partial z} \frac{\theta(H-L)}{\sigma_z^2} e^{\left[\frac{\theta(H-L)}{\sigma_z^2}\left(z-\frac{\theta(H+L)}{2}\right)\right]} dz\right),$$

where we have also used the fact that $\frac{\phi_H(z)}{\phi_L(z)} = exp\left[\frac{\theta(H-L)}{\sigma^2}\left(z - \frac{\theta(H+L)}{2}\right)\right]$. Integrating by parts again, we have

$$\int_{-\infty}^{\bar{z}} \frac{\partial(\phi_H(z))}{\partial z} e^{\left[\frac{\theta(H-L)}{\sigma_z^2}\left(z-\frac{\theta(H+L)}{2}\right)\right]} dz$$

$$= \phi_H(\bar{z}) e^{\left[\frac{\theta(H-L)}{\sigma_z^2}\left(\bar{z}-\frac{\theta(H+L)}{2}\right)\right]} - \frac{\theta(H-L)}{\sigma_z^2} \int_{-\infty}^{\bar{z}} \phi_H(z) e^{\left[\frac{\theta(H-L)}{\sigma_z^2}\left(z-\frac{\theta(H+L)}{2}\right)\right]} dz,$$

and the remaining integral can be evaluated directly by recognizing that its integrand is itself a normal pdf. Note too that $\frac{\phi_H(\bar{z})}{\phi_L(\bar{z})} = exp\left[\frac{\theta(H-L)}{\sigma^2}\left(\bar{z} - \frac{\theta(H+L)}{2}\right)\right] = \frac{A}{B\frac{v}{1-v}}$. Summing up, equation (A.11) can be rewritten as

$$\frac{\partial(E[V(z \mid v \leq \bar{v})])}{\partial\left(\frac{1}{\sigma_z^2}\right)} = \left(2c\left(cR\sigma_y^2 + 1\right)\right)^{-1} A^2 \frac{v^2}{1-v}$$

$$\cdot\left[\frac{1-v}{Av}\phi_H(\bar{z})\frac{\sigma_z^2}{2}2(\bar{z}-\theta L) - \left(\theta(H-L)\right)^2 exp\left[\frac{\theta^2(H-L)^2}{\sigma_z^2}\right]\Phi_{2H-L}(\bar{z})\right.$$

$$-\left(\frac{1-v}{Av}\right)^2 \phi_L(\overline{z})\frac{\sigma_z^2}{2}\left(\overline{z}+\theta(2H-3L)+2\big(\theta(H-L)\big)\right)^2 exp\left[\frac{\theta^2(H-L)^2}{\sigma_z^2}\right]\Phi_{2H-L}(\overline{z})$$

$$-2\frac{1-v}{Av}\phi_H(\overline{z})\frac{\sigma_z^2}{2}(\overline{z}-\theta H) \quad +\left(\frac{1-v}{Av}\right)^2 \phi_L(\overline{z})\frac{\sigma_z^2}{2}(\overline{z}-\theta L)\Bigg]$$

$$=\left(2c\left(cR\sigma_y^2+1\right)\right)^{-1} A^2\frac{v^2}{1-v}\big(\theta(H-L)\big)^2 exp\left[\frac{\theta^2(H-L)^2}{\sigma_z^2}\right]\Phi_{2H-L}(\overline{z})>0 \quad (A.12)$$

which is the desired result.

Regarding sensitivity, we use the facts that $\frac{\partial\phi_a(z)}{\partial\theta}=-a\frac{\partial\phi_a(z)}{\partial z}$ and $\frac{\phi_H(z)^2}{\phi_L(z)}=$ $exp\left[\frac{(\theta(H-L))^2}{\sigma^2}\right]\phi_{2H-L}(z)$. We have

$$\frac{\partial(E[V(z \mid v\leq\overline{v})])}{\partial\theta}$$

$$=\left(2c\left(cR\sigma_y^2+1\right)\right)^{-1}$$

$$\cdot\left[A^2\frac{v^2}{1-v}\left(exp\left[\frac{\big(\theta(H-L)\big)^2}{\sigma_z^2}\right]\int_{-\infty}^{\overline{z}}\frac{\partial\phi_{2H-L}(z)}{\partial\theta}dz+\frac{2\theta(H-L)^2}{\sigma_z^2}exp\left[\frac{\big(\theta(H-L)\big)^2}{\sigma_z^2}\right]\Phi_{2H-L}(\overline{z})\right)\right.$$

$$\left.-2Av\int_{-\infty}^{\overline{z}}\frac{\partial\big(\phi_H(z)\big)}{\partial\theta}dz \quad +(1-v)\int_{-\infty}^{\overline{z}}\frac{\partial\big(\phi_L(z)\big)}{\partial\theta}dz\right]$$

$$=\left(2c\left(cR\sigma_y^2+1\right)\right)^{-1}A^2\frac{v^2}{1-v}$$

$$\cdot\left[exp\left[\frac{\big(\theta(H-L)\big)^2}{\sigma_z^2}\right](-2H\right.$$

$$+L)\phi_{2H-L}(\overline{z}) \quad \frac{2\theta(H-L)^2}{\sigma_z^2}exp\left[\frac{\big(\theta(H-L)\big)^2}{\sigma_z^2}\right]$$

$$\left.-2\frac{1-v}{Av}(-H)\phi_H(\overline{z})+\left(\frac{1-v}{Av}\right)^2(-L)\phi_L(\overline{z})\right]$$

$$= \left(2c\left(cR\sigma_y^2 + 1\right)\right)^{-1} A^2 \frac{v^2}{1-v} \frac{2\theta(H-L)^2}{\sigma_z^2} exp\left[\frac{\left(\theta(H-L)\right)^2}{\sigma_z^2}\right] \Phi_{2H-L}(\overline{z}) > 0.$$

PROOF OF LEMMA 3.1 (Solution to the Principal's Problem)

Given α and β, the agent's problem is effectively to choose an effort level to maximize his certainty equivalent

$$CE\left(w\left(a, y(e,a)\right)\right) = \alpha + \beta(\lambda a + e) - \frac{R\beta^2\sigma_y^2}{2} - \frac{ce^2}{2}, a \in \{H, L\}. \qquad (A.13)$$

The solution to the effort problem for a particular a is then

$$e(a) = \frac{\beta_a\gamma}{c}, a \in \{H, L\}. \qquad (A.14)$$

Note that this result holds only for $\beta \geq 0$; otherwise, the solution is simply $e(a) = 0$, $a \in \{H,L\}$. The individual rationality (IR) constraint for the L type agent binds, so that

$$\alpha_L + \beta_L\left(\lambda L + e(L)\right) - \frac{R\beta_L^2\sigma_y^2}{2} - \frac{ce(L)^2}{2} = r_0. \qquad (A.15)$$

Given this, we can express α_L as

$$\alpha_L = r_0 - \beta_L\left(\lambda L + e(L)\right) + \frac{R\beta_L^2\sigma_y^2}{2} + \frac{ce(L)^2}{2}. \qquad (A.16)$$

The truth-telling (TT) constraint of the H type agent also binds, so that

$$\alpha_H + \beta_H\left(\lambda H + e(H)\right) - \frac{R\beta_H^2\sigma_y^2}{2} - \frac{ce(H)^2}{2}$$

$$= \alpha_L + \beta_L\left(\lambda H + e(L)\right) - \frac{R\beta_L^2\sigma_y^2}{2} - \frac{ce(L)^2}{2}. \qquad (A.17)$$

Given this, we can express α_H as

$$\alpha_H = \alpha_L + \beta_L\left(\lambda H + \gamma e(L)\right) - \frac{R\beta_L^2\sigma_y^2}{2} - \frac{ce(L)^2}{2}$$

$$-\beta_H\left(\lambda H + \gamma e(H)\right) + \frac{R\beta_H^2\sigma_y^2}{2} + \frac{ce(H)^2}{2}. \qquad (A.18)$$

The principal's objective function is

$$\max_{\{\beta_H,\beta_L,e(a)\}} x\left[\lambda H + e(H) - \left(\alpha_H + \beta_H(\lambda H + \gamma e(H))\right)\right]$$

$$+ (1-x)\left[\lambda L + e(L)\right.$$

$$\left.- \left(\alpha_L + \beta_L(\lambda L + e(L))\right)\right]. \qquad (A.19)$$

Combining equations (A.17), (A.18), and (A.19) allows us to solve for the optimal values of β_H and β_L. Note that it is here that the non-negativity constraint on effort (in particular, on β_L) gives rise to the threshold. Plugging those values back in to equations (A.15) and (A.16) immediately gives us the optimal values of α_H and α_L.

Further, plugging these optimal values in to equations (A.13) and (A.14) and taking the expectation over ε gives us the expected compensation for each type; and subtracting this expected compensation from expected production gives the expressions for expected profit by type. Finally, taking the expected value of profit over the agent types gives equation (3.16).

PROOF OF PROPOSITION 3.1 (Aggregation of Information)

Recall that the information influences the menu of contracts only through its effect on the principal's posterior beliefs, and further, that this effect is determined by the likelihood ratio. The ratio of conditional joint densities of any distribution meeting Assumption 3.2 can in general be written as

$$\frac{g_H(z_1,\cdots,z_n)}{g_L(z_1,\cdots,z_n)} = \frac{\exp\left[H\sum_{i=1}^n p_i z_i + \sum_{i=1}^n d_i(H) + S(z_1,z_2,\cdots,z_n)\right]}{\exp\left[L\sum_{i=1}^n p_i z_i + \sum_{i=1}^n d_i(L) + S(z_1,z_2,\cdots,z_n)\right]}$$

$$= \exp\left[(H-L)\sum_{i=1}^n p_i z_i +\right]\exp\left[\sum_{i=1}^n (d_i(H) - d_i(L))\right],$$

and we see immediately that the optimal menu of contracts depends only on a linear aggregation of the information $z_1, \cdots, z_n$ using weights $p_1, \cdots, p_n$, respectively.

PROOF OF PROPOSITION 3.2 (Optimal Relative Weights in Adverse Selection, Uncorrelated)

We show the derivation for the case of uncorrelated information. The derivation for correlated information is similar. The joint conditional density for uncorrelated information meeting Assumption 3.2 can be written as $g_a(z_1, z_2) = \exp[p_1 a z_1 + p_2 a z_2 - d_1(a) - d_2(a) + s_1(z_1) + s_2(z_2)]$. Since z_1 and z_2 are independent, by factorization we can express the density function for z_1 and z_2 as

$$g_a(z_1) = \exp[p_1 a z_1 - d_1(a) + s_1(z_1)];$$

$$g_a(z_2) = \exp[p_2 a z_2 - d_2(a) + s_2(z_2)].$$

One can show that

$$\frac{\partial E[w(H)]}{\partial z_i} = -\frac{v}{1-v}\frac{c\lambda^2(H-L)^2}{cR\sigma^2+1}\frac{\partial LR(a; z_1, z_2)}{\partial z_i};$$

$$\frac{\partial E[w(L)]}{\partial z_i} = -\frac{\lambda\left(\gamma^2 - c\lambda(H-L)\frac{u}{1-u}\right)}{(cR\sigma^2+1)}(H-L)\frac{v}{1-v}\frac{\partial LR(a; z_1, z_2)}{\partial z_i}.$$

Proceeding,

$$\frac{\partial E[w(H; z_1, z_2)]/\partial z_1}{\partial E[w(H; z_1, z_2)]/\partial z_2} = \frac{\frac{\partial LR(a; z_1, z_2)}{\partial z_1}}{\frac{\partial LR(a; z_1, z_2)}{\partial z_2}} = \frac{p_1}{p_2};$$

$$\frac{\partial E[w(L; z_1, z_2)]/\partial z_1}{\partial E[w(L; z_1, z_2)]/\partial z_2} = \frac{\frac{\partial LR(a; z_1, z_2)}{\partial z_1}}{\frac{\partial LR(a; z_1, z_2)}{\partial z_2}} = \frac{p_1}{p_2}.$$

An implication is that

$$\frac{\partial E[w(a; z_1, z_2)]/\,\partial z_1}{\partial E[w(a; z_1, z_2)]/\,\partial z_2} = \frac{p_1}{p_2}.$$

Then adopt the technique used in two-sided Laplace transform theory to derive expressions for $E(z_1)$ and $Var(z_1)$. Since $\int g_a(z_1)dz_1 = 1$, we must have

$$\int \exp[p_1 a z_1 + s_1(z_1)]\, dz_1 = \exp[d_1(a)].$$

Therefore, on differentiating with respect to $p_1 a$, we have

$$\int z_1 \exp[p_1 a z_1 + s_1(z_1)]\, dz_1 = d_1'(a) \exp[d_1(a)]\,/p_1, \qquad (A.20)$$

and rearranging the equation we have

$$\int z_1 \exp[p_1 a z_1 - d_1(a) + s_1(z_1)]\, dz_1 = d_1'(a)/p_1.$$

Thus, we have $E(z_1) = d_1'(a)/p_1$. To obtain the expression for $E(z_1^2)$, we differentiate equation (A.20) with respect to $p_1 a$. We have

$$\int z_1^2 \exp[p_1 a z_1 + s_1(z_1)]\, dz_1 = \exp[d_1(a)]\left[d_1''(a) + d_1'(a)^2\right]/p_1^2.$$

Therefore, we have

$$E(z_1^2) = \left[d_1''(a) + d_1'(a)^2\right]/p_1^2.$$

Invoking the relationship between the moments,

$$Var(z_1) = E(z_1^2) - E(z_1)^2 = d_1''(a)/p_1^2.$$

In addition, differentiating $E(z_1)$ with respect to a, we obtain

$$\frac{\partial E(z_1)}{\partial a} = \frac{d_1''(a)}{p_1} = \frac{Var(z_1)}{p_1}. \qquad (A.21)$$

Rearranging the above equation (A.21), we have

$$\frac{\partial E(z_1)/\,\partial a}{Var(z_1)} = p_1.$$

Similarly, we can obtain

$$\frac{\partial E(z_2)/\partial a}{Var(z_2)} = p_2.$$

Plugging the results back to the relative weights, we have

$$\frac{\partial w(a; z_1, z_2)/\ \partial z_1}{\partial w(a; z_1, z_2)/\ \partial z_2} = \frac{\partial E(z_1)/\ \partial a}{\partial E(z_2)/\ \partial a}\frac{1/Var(z_1)}{1/Var(z_2)}.$$

PROOF OF PROPOSITION 3.3 (Optimal Relative Weights in Adverse Selection, Correlated)

The joint conditional density for correlated information meeting Assumption 3.2 can be written as $g_a(z_1, z_2) = \exp[p_1 a z_1 + p_2 a z_2 - d_1(a) - d_2(a) + s_1(z_1 - \kappa z_2)]$. We begin by making the linear transformation $z_3 = z_1 - \kappa z_2$, so that

$$E(z_3) = E(z_1) - \kappa E(z_2),$$

and

$$Var(z_3) = Var(z_1) + \kappa^2 Var(z_2) - 2kCov(z_1, z_2). \qquad (A.22)$$

Note that the joint density function of z_2 and z_3 is $g_a(z_2, z_3) = \exp[p_1 a z_3 + (kp_1 a + p_2 a)z_2 - d_1(a) - d_2(a) + s_1(z_3)]$. It is evident that z_2 and z_3 are independent and their joint density function belongs to the class considered in Assumption 3.2.

Since $z_1 = z_3 + \kappa z_2$, we have

$$Var(z_1) = Var(z_3) + \kappa^2 Var(z_2). \qquad (A.23)$$

Combining equation (A.22) and equation (A.23), we have

$$\kappa = \frac{Cov(z_1, z_2)}{Var(z_2)}.$$

Applying the result for uncorrelated signals from Proposition 3.2, for an agent of type a, we have

$$\frac{\partial E[w(a;z_2,z_3)]/\,\partial z_2}{\partial E[w(a;z_2,z_3)]/\,\partial z_3}=\frac{\partial E(z_2)/\,\partial a}{\partial E(z_3)/\,\partial a}\frac{1/Var(z_2)}{1/Var(z_3)}.$$

We also have

$$\frac{\partial E[w(a;z_2,z_3)]/\,\partial z_2}{\partial E[w(a;z_2,z_3)]/\,\partial z_3}=\frac{\partial LR(a;z_2,z_3)/\,\partial z_2}{\partial LR(a;z_2,z_3)/\,\partial z_3}=\kappa+\frac{p_2}{p_1}.$$

We can also show that

$$\frac{\partial E[w(a;z_1,z_2)]/\partial z_1}{\partial E[w(a;z_1,z_2)]/\partial z_2}=\frac{\partial LR(a;z_1,z_2)/\partial z_1}{\partial LR(a;z_1,z_2)/\partial z_2}=\frac{p_2}{p_1}.$$

Thus, the relative weight is

$$\frac{\partial E[w(a;z_1,z_2)]/\partial z_1}{\partial E[w(a;z_1,z_2)]/\partial z_2}=\frac{\partial E(z_2)/\partial a}{\partial E(z_3)/\partial a}\frac{1/Var(z_2)}{1/Var(z_3)}-k$$

$$=\frac{\partial \mathrm{E}(\mathrm{z}_1)/\,\partial \mathrm{a}-\frac{\mathrm{cov}}{\mathrm{Var}(\mathrm{z}_2)}\partial \mathrm{E}(\mathrm{z}_2)/\,\partial \mathrm{a}}{\partial \mathrm{E}(\mathrm{z}_2)/\,\partial \mathrm{a}-\frac{\mathrm{cov}}{\mathrm{Var}(\mathrm{z}_1)}\partial \mathrm{E}(\mathrm{z}_1)/\,\partial \mathrm{a}}\frac{1/\mathrm{Var}(\mathrm{z}_1)}{1/\mathrm{Var}(\mathrm{z}_2)},$$

which is the desired result.

Proof of Lemma 4.1& 4.2:

Suppose the principal implements effort control such that she hires a self-interested agent and contracts only on the signal of effort. The agent observes the cost characteristic k, but not the outcome noise, so the agent's effort problem is to maximize his expected payoff with expectation taken over outcome noise:

$$E(U^A)=\int_{-\infty}^{+\infty}-exp\{-R\left[\alpha_O+\beta_O(\theta e_O+\varepsilon)-\frac{e_O^2}{2}+ke_O\right\}f(\varepsilon)d\varepsilon$$

with respect to e_O. Since ε is normal distribution and $f(\varepsilon)=\frac{1}{\sigma_\varepsilon\sqrt{2\pi}}exp(-\frac{\varepsilon}{\sigma_\varepsilon^2})$, by completing the square, we have

$$E(U^A)=-exp\{-R\left[\alpha_O+\beta_O\theta e_O-\frac{e_O^2}{2}+ke_O-\frac{R\beta_O^2\sigma_\varepsilon^2}{2}\right\}$$

This is equivalent to maximizing the standard certainty equivalent:

$$CE = \alpha_O + \beta_O(\theta e_O) - (R/2)\beta_O^2\sigma_\varepsilon^2 - e_O^2/2 + ke_O$$

with respect to e_O.

FOC(e_O) yields the effort policy[58]:

$$e_O = \theta\beta_O + k. \tag{A.24}$$

The second-order condition (SOC) is satisfied because the effort cost function is convex.

The agent's individual rationality constraint can be formulated as: [59]

$$\iint -exp\{-R\left[\alpha_O + \beta_O(\theta e_O + \varepsilon) - \frac{e_O^2}{2} + ke_O\right\} f(\varepsilon)g(k)d\varepsilon dk \geq -exp(-Rr_0). \tag{A.25}$$

Since $\varepsilon \sim N(0,\sigma_\varepsilon^2)$, $k \sim N(0,\sigma_k^2)$, and they are independent, we can first integrate (A.25) over outcome noise,

$$\int_{-\infty}^{+\infty} -exp\{-R[(\alpha_O + \beta_O\theta e_O) - \frac{R\beta_O^2\sigma_\varepsilon^2}{2} - \frac{e_O^2}{2} + ke_O]\}g(k)dk \geq -exp(-Rr_0) \tag{A.26}$$

Substituting the optimal effort solution (A.24) to (A.26), the IR constraint then is

$$-exp[-R(\alpha_O - \frac{R\beta_O^2\sigma_\varepsilon^2}{2})]\int_{-\infty}^{+\infty} exp[-R\frac{(\theta\beta_O + k)^2}{2}]\,g(k)dk \geq -exp(-Rr_0).$$

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[58] For the ease of tractability, we assume normality for k. We justify this by using the arguments in most of the outcome function in agency models, that is, it is negligible of encountering a sufficiently negative k. Furthermore, we also derive the analytical expression of the IR constraint under half-normal distribution for k, which essentially removes the concern for having a negative effort. The comparative statics for the LHS of the IR constraint with respect to the model parameters are the same for half-normal and normal distribution, suggesting that the results of our model is not from the symmetry of the k distribution. But solution for the principal's expected utility is not tractable under half-normal.

[59] r_0 is later normalized to be 0 without loss of generality.

Since k is normal distribution and $g(k) = \frac{1}{\sigma_k\sqrt{2\pi}} exp(-\frac{k^2}{2\sigma_k^2})$, by completing the square, we have

$$\int_{-\infty}^{+\infty} exp\{-R\frac{(\theta\beta_O+k)^2}{2}\}\, g(k)dk = exp\{-\frac{R\theta^2\beta_O^2}{2(1+R\sigma_k^2)}\}/\sqrt{1+R\sigma_k^2}.$$

Applying (A.24), the IR constraint becomes

$$-exp\{-R[\alpha_O - \frac{R\beta_O^2\sigma_\varepsilon^2}{2} + \frac{\theta^2\beta_O^2}{2(1+R\sigma_k^2)}]\}/\sqrt{1+R\sigma_k^2} \geq -exp(-Rr_0).$$

We can rewrite the IR constraint in its certainty equivalent form:[60]

$$\alpha_O - \frac{R\beta_O^2\sigma_\varepsilon^2}{2} + \frac{\theta^2\beta_O^2}{2(1+R\sigma_k^2)} + \frac{\mathrm{Log}\left(\sqrt{1+R\sigma_k^2}\right)}{R} \geq r_0.$$

The agent's individual rationality constraint binds:

$$\alpha_O - \frac{R\beta_O^2\sigma_\varepsilon^2}{2} + \frac{\theta^2\beta_O^2}{2(1+R\sigma_k^2)} + \frac{\mathrm{Log}\left(\sqrt{1+R\sigma_k^2}\right)}{R} = r_0. \qquad (A.27)$$

The objective of the principal is to maximize her expected payoff with expectations taken over both outcome noise and cost characteristic,

$$\max_{\alpha_O,\beta_O} \iint \{\theta e_y + \varepsilon - [\alpha_O + \beta_O(\theta e_O + \varepsilon)]\}\, d\varepsilon dk. \qquad (A.28)$$

Applying equation (A.24) and (A.27) to the objective of the principal (A.28) and notice that E(k)=0, the principal's problem becomes

$$\max_{\beta_O}\left\{\theta^2\beta_O - \theta^2\beta_O^2 - (R/2)\beta_O^2(\sigma_\varepsilon^2) + \frac{\theta^2\beta_O^2}{2(1+R\sigma_k^2)} + \frac{\mathrm{Log}\left(\sqrt{1+R\sigma_k^2}\right)}{R} - r_0\right\}. \quad (A.29)$$

The SOC (Second Order Condition) of the principal's mechanism design problem is satisfied.

[60] Notice that the certainty equivalent of the IR constraint is different from the CE of the agent's utility in (A4).

FOC(β_O) of (A.29) yields the coefficient on outcome y:

$$\beta_O = \frac{\theta^2}{\theta^2 + \theta^2 \dfrac{R\sigma_k^2}{1 + R\sigma_k^2} + R\sigma_\varepsilon^2}. \qquad (A.30)$$

One can easily show that the coefficient on outcome y under the standard moral hazard model (assuming k is observable) is

$$\tilde{\beta}_O = \frac{\theta^2}{\theta^2 + R\sigma_\varepsilon^2}. \qquad (A.31)$$

Comparing (A.30) and (A.31), we can observe that β_O converges to $\tilde{\beta}_O$ as σ_k^2 converges to 0.

Applying (A.30) to the agent's effort policy, the variance of the optimal effort is

$$Var(e_O) = Var(k)$$

Applying the coefficient on outcome to the principal's objective function (A.29), the expected payoff of the principal at optimum is

$$\pi_O = \frac{\theta^4}{2\left(\theta^2 + \theta^2 \dfrac{R\sigma_k^2}{1 + R\sigma_k^2} + R\sigma_\varepsilon^2\right)} + \frac{\mathrm{Log}(1 + R\sigma_k^2)}{2R}. \qquad (A.32)$$

Suppose the principal implements effort control such that she hires a self-interested agent and contracts only on the signal of effort. The agent observes the cost characteristic k, but not the outcome noise, so the agent's effort problem is to maximize his expected payoff with expectation taken over outcome noise:

$$E(U^A) = \int_{-\infty}^{+\infty} - exp\{-R\left[\alpha_B + \beta_B(e_B + \eta) - \frac{e_B^2}{2} + ke_B\right\} f(\eta)d\eta.$$

with respect to e_B. Since η is normal distribution and $f(\eta) = \frac{1}{\sigma_\eta\sqrt{2\pi}} exp(-\frac{\eta^2}{\sigma_\eta^2})$, by completing the square, we have

$$E(U^A) = -exp\{-R\left[\alpha_B + \beta_B e_B - \frac{e_B^2}{2} + ke_B - \frac{R\beta_B^2\sigma_\eta^2}{2}\right\}.$$

This is equivalent to maximizing the standard certainty equivalent:

$$CE = \alpha_B + \beta_B(e_B) - (R/2)\beta_B^2\sigma_\eta^2 - e_B^2/2 + ke_B.$$

with respect to e_B.

FOC(e_B) yields the effort policy:

$$e_B = \beta_B + k. \tag{A.33}$$

The second-order condition (SOC) is satisfied because the effort cost function is convex.

The agent's individual rationality constraint can be formulated as:

$$\iint -exp\{-R\left[\alpha_z + \beta_z(e_z + \eta) - \frac{e_z^2}{2} + ke_z\right\} f(\eta)g(k)d\eta dk \geq -exp(-Rr_0).$$

Since $\eta \sim N(0, \sigma_\eta^2)$, $k \sim N(0, \sigma_k^2)$, and they are independent, we can first integrate over outcome noise,

$$\int -exp\{-R[(\alpha_B + \beta_B e_B) - \frac{R\beta_B^2\sigma_\eta^2}{2} - \frac{e_B^2}{2} + ke_B]\}g(k)dk \geq -exp(-Rr_0).$$

Substituting the optimal effort solution, the IR constraint then is

$$-exp[-R(\alpha_B - \frac{R\beta_B^2\sigma_\eta^2}{2})]\int_{-\infty}^{+\infty} exp[-R\frac{(\beta_B+k)^2}{2}]\,g(k)dk \geq -exp(-Rr_0).$$

Since k is normal distribution and $g(k) = \frac{1}{\sigma_k\sqrt{2\pi}}exp(-\frac{k^2}{\sigma_k^2})$, by completing the square, we have

$$\int_{-\infty}^{+\infty} exp\{-R\frac{(\beta_B + k)^2}{2}\}g(k)dk = exp\{-\frac{R\beta_B^2}{2(1 + R\sigma_k^2)}\}/\sqrt{1 + R\sigma_k^2}.$$

Then IR constraint becomes

$$-exp\{-R[\alpha_B - \frac{R\beta_B^2\sigma_\eta^2}{2} + \frac{\beta_B^2}{2(1+R\sigma_k^2)}]\}/\sqrt{1 + R\sigma_k^2} \geq -exp(-Rr_0).$$

We can rewrite the IR constraint in its certainty equivalent form:

$$\alpha_B - \frac{R\beta_B^2\sigma_\eta^2}{2} + \frac{\beta_B^2}{2(1+R\sigma_k^2)} + \frac{\mathrm{Log}(1+R\sigma_k^2)}{2R} \geq r_0.$$

The agent's individual rationality constraint binds:

$$\alpha_B - \frac{R\beta_B^2\sigma_\eta^2}{2} + \frac{\beta_B^2}{2(1+R\sigma_k^2)} + \frac{\mathrm{Log}(1+R\sigma_k^2)}{2R} = r_0.$$

The objective of the principal is to maximize her expected payoff with expectations taken over both outcome noise and cost characteristic,

$$\max_{\alpha_B,\beta_B} \iint \{\theta e_B + \varepsilon - [\alpha_B + \beta_B(e_B+\eta)]\}\, d\eta dk.$$

Applying equation (A23) and (A30) to the objective of the principal (A31) and notice that E(k)=0, the principal's problem becomes

$$\max_{\beta_B} \left\{\theta\beta_B - \beta_B^2 - (R/2)\beta_B^2(\sigma_\eta^2) + \frac{\beta_B^2}{2(1+R\sigma_k^2)} + \frac{\mathrm{Log}(1+R\sigma_k^2)}{2R} - r_0\right\}.$$

The SOC (Second Order Condition) of the principal's mechanism design problem is satisfied.

FOC(β_B) of (A32) yields the coefficient on the signal *z*:

(A33) $\beta_B = \frac{\theta}{1+\frac{R\sigma_k^2}{1+R\sigma_k^2}+R\sigma_\eta^2}$.

One can easily show that the coefficient on signal *z* under the standard moral hazard model (assuming k is observable) is

(A34) $\tilde{\beta}_B = \frac{\theta}{1+R\sigma_\eta^2}$.

Comparing (A33) and (A34), we can observe that (A33) converges to (A4) as σ_k^2 converges to 0.

Applying the solution to the agent's effort policy, the variance of the optimal effort is

(A35) $Var(e_B) = Var(k)$.

Applying the optimal coefficient on the signal of effort to the principal's objective function, the expected payoff of the principal at optimum is:

$$\pi_B = \frac{\theta^2}{2\left(1 + \frac{R\sigma_k^2}{1 + R\sigma_k^2} + R\sigma_\eta^2\right)} + \frac{\mathrm{Log}(1 + R\sigma_k^2)}{2R}.$$

Suppose the principal implements clan control such that she offers the agent a fixed wage. The expected payoff of the principal at the optimum is

$$\pi_C = \frac{\theta^4}{2(2r - \theta^2 + R\sigma_\varepsilon^2)} + \frac{\sigma_k^2}{2}.$$

The agent's expected utility is

$$E(U^A) = -exp\{-R\left[\alpha_C + \lambda(\theta e_C + \varepsilon) - \frac{e_C^2}{2} + ke_C\right\}.$$

The agent's effort problem is to maximize the certainty equivalent:

$$CE = \lambda\theta e_C + \alpha_C - \frac{R\lambda^2\sigma_\varepsilon^2}{2} - e_C^2/2 + ke_C.$$

with respect to e_C.

FOC(e_C) yields the effort policy:

$$e_C = \lambda\theta + k.$$

The second-order condition (SOC) is satisfied because the effort cost function is convex.

The individual rationality constraint is:

$$-exp\{-R[\lambda(\theta e_C) + \alpha_C - \frac{R\lambda^2\sigma_\varepsilon^2}{2} - e_C^2/2 + ke_C]\} \geq -exp(-Rr_0), \text{ for all } k.$$

Applying effort policy,

$$-exp\{-R[\alpha_C - \frac{R\lambda^2\sigma_\varepsilon^2}{2} + (\lambda\theta + k)^2/2)]\} \geq -exp(-Rr_0), \text{ for all } k.$$

At the optimum, the IR constraint binds,

$$\alpha_C - \frac{R\lambda^2\sigma_\varepsilon^2}{2} + (\lambda\theta + k)^2/2 = 0.$$

Therefore, the fixed wage at the optimum is

$$\alpha_C = \frac{R\lambda^2\sigma_\varepsilon^2}{2} - (\lambda\theta + k)^2/2.$$

This fixed wage is paid to the agent at the end of the period when the agent has revealed his information on task characteristics to the principal through his actions in the socialization process.

The expected agent's salary pre-contracting is then

$$E(\alpha_C) = \frac{R\lambda^2\sigma_\varepsilon^2}{2} - \frac{\lambda^2\theta^2}{2} - \frac{\sigma_k^2}{2}.$$

The variance of the optimal effort is

$$Var(e_C) = Var(k).$$

The principal needs to choose the optimal level of alignment for the agent after the agent accepts the contract. It suggests that the optimal level of alignment can't depend on the cost characteristic. Hence the principal's objective is to maximize her expected payoff with expectation taken with respect to both outcome noise and cost characteristic. The expected profit of the principal is given by

$$\iint [\theta e_C + \varepsilon - \alpha_C - r\lambda^2] f(\varepsilon) g(k) d\varepsilon dk.$$

Applying effort policy and the agent's optimal compensation, the objective of the principal is then

$$\max_{\lambda} \lambda\theta^2 - \frac{R\lambda^2\sigma_\varepsilon^2}{2} + \frac{\lambda^2\theta^2}{2} + \frac{\sigma_k^2}{2} - r\lambda^2.$$

The FOC(λ) yields the optimal degree of alignment

$$\lambda = \frac{\theta^2}{2r - \theta^2 + R\sigma_\varepsilon^2}.$$

The SOC condition is then

$$2r - \theta^2 + R\sigma_\varepsilon^2 \geq 0.$$

The expected payoff of the principal is then

$$\pi_C = \frac{\theta^4}{2(2r - \theta^2 + R\sigma_\varepsilon^2)} + \frac{\sigma_k^2}{2}.$$

PROOF of THEOREM 4.1:

We need to find a pair $(\bar{\sigma}_\varepsilon^2, \bar{\sigma}_k^2)$ with nonnegative values that satisfy the following equations,

$$\pi_O(\sigma_\varepsilon^2, \sigma_k^2) = \pi_B(\sigma_\varepsilon^2, \sigma_k^2)$$

and

$$\pi_O(\sigma_\varepsilon^2, \sigma_k^2) = \pi_C(\sigma_\varepsilon^2, \sigma_k^2).$$

From the first equation, we immediately have

$$\bar{\sigma}_\varepsilon^2 = \theta^2 \sigma_\eta^2.$$

To see the existence of $\bar{\sigma}_k^2$, we construct the following functions.

$$\pi_O(t) = \frac{\theta^4}{2\left(\theta^2 + \theta^2 \frac{Rt}{1+Rt} + R\sigma_\varepsilon^2\right)} + \frac{\mathrm{Log}(1+Rt)}{2R};$$

$$\pi_B(t) = \frac{\theta^2}{2\left(1 + \frac{Rt}{1+Rt} + R\sigma_\eta^2\right)} + \frac{\mathrm{Log}(1+Rt)}{2R};$$

$$\pi_C(t) = \frac{\theta^4}{2(2r - \theta^2 + R\sigma_\varepsilon^2)} + \frac{t}{2}.$$

Next we will show below that $\pi_O(t)$ and $\pi_C(t)$ have either one intersection or no intersection point depending on the value of other parameters.

Scenario 1: if $r < \theta^2$, then no intersection. We construct the following function$\pi(t) = \pi_C(t) - \pi_O(t)$. It is easy to see that $\pi(0) > 0$. Next, we examine

$$\pi'(t) = \frac{d\pi(t)}{dt} = \frac{1}{2} - \frac{1}{2(1+Rt)} + \frac{\theta^6 R}{2\left(\theta^2 + \theta^2 \frac{Rt}{1+Rt} + R\sigma_\varepsilon^2\right)^2 (1+Rt)^2} > 0. \quad (A.35)$$

Therefore, for all $\tilde{t} \geq 0$, we have $\pi(\tilde{t}) > \pi(0) > 0$, which suggests no intersection between $\pi_O(t)$ and $\pi_C(t)$. Obviously, clan control dominates outcome control in this scenario. And obviously there are unlimited pairs of $(\sigma_\varepsilon^2, \sigma_k^2)$ such that $\pi_B(\sigma_\varepsilon^2, \sigma_k^2) = \pi_C(\sigma_\varepsilon^2, \sigma_k^2)$.

Scenario 2: if $r = \theta^2$, then there is only one intersection. Based on the analysis in scenario 1, we can see that $\pi_O(t)$ and $\pi_C(t)$ only intersect at $t = 0$.

Scenario 3: if $r > \theta^2$, then there is only one intersection. First, it is obvious that $\pi(0) < 0$. Then we examine the following expression by using the L'Hospital's rule,

$$\lim_{t\to+\infty} \frac{\pi_O(t)}{\pi_C(t)} = \lim_{t\to+\infty} \frac{\frac{\theta^4}{2\left(\theta^2 + \theta^2 \frac{Rt}{1+Rt} + R\sigma_\varepsilon^2\right)} + \frac{\mathrm{Log}(1+Rt)}{2R}}{\frac{\theta^4}{2(2r - \theta^2 + R\sigma_\varepsilon^2)} + \frac{t}{2}}$$

$$= \lim_{t\to+\infty} \frac{\frac{\mathrm{Log}(1+Rt)}{2R}}{\frac{\theta^4}{2(2r - \theta^2 + R\sigma_\varepsilon^2)} + \frac{t}{2}} = \lim_{t\to+\infty} \frac{R}{1+Rt} = 0.$$

Hence, $\pi(\tilde{t}) > 0$ for a sufficiently large $\tilde{t} > 0$. Then by invoking the intermediate value theorem, $\pi_O(t)$ and $\pi_C(t)$ have at least one intersection. Next suppose there are two intersections at t_1 and t_2, which means $\pi(t_1) = 0$ and $\pi(t_2) = 0$, then by Rolle's theorem there exists at least on $\bar{t}$ between t_1 and t_2 such that $\pi'(\bar{t}) = 0$. However, as is already shown in (A.35), we observe that $\pi'(t) > 0$ for all $t > 0$. Hence, there is only one intersection.

To summarize, if $r \geq \theta^2$, then there exists nonnegative values of $(\bar{\sigma}_\varepsilon^2, \bar{\sigma}_k^2)$ such that $\pi_O(\bar{\sigma}_\varepsilon^2, \bar{\sigma}_k^2) = \pi_B(\bar{\sigma}_\varepsilon^2, \bar{\sigma}_k^2) = \pi_C(\bar{\sigma}_\varepsilon^2, \bar{\sigma}_k^2)$; if $\theta^2/2 < r < \theta^2$, then there are unlimited pairs of $(\sigma_\varepsilon^2, \sigma_k^2)$ such that,

$$\pi_B(\bar{\sigma}_\varepsilon^2, \bar{\sigma}_k^2) = \pi_C(\bar{\sigma}_\varepsilon^2, \bar{\sigma}_k^2) > \pi_O \bar{\sigma}_\varepsilon^2, \bar{\sigma}_k^2).$$

PROOF of THEOREM 4.2:

It is obvious that point O is the intersection of the three contour lines OA, OB, and OC. Proof of Theorem 1 shows that,

$$\bar{\sigma}_\varepsilon^2 = \theta^2 \sigma_\eta^2,$$

which suggests that OA must be a straight line.

To see the slope of the contour line OB is always positive, we apply the rule of implicit differentiation to the following equation,

$$\pi_B(\sigma_\varepsilon^2, \sigma_k^2) = \pi_C(\sigma_\varepsilon^2, \sigma_k^2),$$

such that,

$$\frac{\partial \pi_B(\sigma_\varepsilon^2, \sigma_k^2)}{\partial \sigma_\varepsilon^2} \frac{d\sigma_\varepsilon^2}{d\sigma_k^2} + \frac{\partial \pi_B(\sigma_\varepsilon^2, \sigma_k^2)}{\partial \sigma_k^2} = \frac{\partial \pi_C(\sigma_\varepsilon^2, \sigma_k^2)}{\partial \sigma_\varepsilon^2} \frac{d\sigma_\varepsilon^2}{d\sigma_k^2} + \frac{\partial \pi_C(\sigma_\varepsilon^2, \sigma_k^2)}{\partial \sigma_k^2}.$$

Therefore, we have

$$\frac{d\sigma_\varepsilon^2}{d\sigma_k^2} = \frac{\left(\frac{\partial \pi_C(\sigma_\varepsilon^2, \sigma_k^2)}{\partial \sigma_k^2} - \frac{\partial \pi_B(\sigma_\varepsilon^2, \sigma_k^2)}{\partial \sigma_k^2}\right)}{\frac{\partial \pi_B(\sigma_\varepsilon^2, \sigma_k^2)}{\partial \sigma_\varepsilon^2} - \frac{\partial \pi_C(\sigma_\varepsilon^2, \sigma_k^2)}{\partial \sigma_\varepsilon^2}}.$$

It is straightforward to notice that $\frac{\partial \pi_B(\sigma_\varepsilon^2, \sigma_k^2)}{\partial \sigma_\varepsilon^2} = 0$, $\frac{\partial \pi_C(\sigma_\varepsilon^2, \sigma_k^2)}{\partial \sigma_\varepsilon^2} < 0$ and $\frac{\partial \pi_C(\sigma_\varepsilon^2, \sigma_k^2)}{\partial \sigma_k^2} - \frac{\partial \pi_B(\sigma_\varepsilon^2, \sigma_k^2)}{\partial \sigma_k^2} > 0$ (Please refers to equation A.35 for the proof). Hence, we have $\frac{d\sigma_\varepsilon^2}{d\sigma_k^2} > 0$.

To see the slope of the contour line OC is always negative, we follow the same procedure as in the above scenario.

$$\frac{\partial \pi_O(\sigma_\varepsilon^2, \sigma_k^2)}{\partial \sigma_\varepsilon^2}\frac{d\sigma_\varepsilon^2}{d\sigma_k^2} + \frac{\partial \pi_O(\sigma_\varepsilon^2, \sigma_k^2)}{\partial \sigma_k^2} = \frac{\partial \pi_C(\sigma_\varepsilon^2, \sigma_k^2)}{\partial \sigma_\varepsilon^2}\frac{d\sigma_\varepsilon^2}{d\sigma_k^2} + \frac{\partial \pi_C(\sigma_\varepsilon^2, \sigma_k^2)}{\partial \sigma_k^2}.$$

Proceeding, we have

$$\frac{d\sigma_\varepsilon^2}{d\sigma_k^2} = \frac{\left(\frac{\partial \pi_C(\sigma_\varepsilon^2, \sigma_k^2)}{\partial \sigma_k^2} - \frac{\partial \pi_O(\sigma_\varepsilon^2, \sigma_k^2)}{\partial \sigma_k^2}\right)}{\frac{\partial \pi_O(\sigma_\varepsilon^2, \sigma_k^2)}{\partial \sigma_\varepsilon^2} - \frac{\partial \pi_C(\sigma_\varepsilon^2, \sigma_k^2)}{\partial \sigma_\varepsilon^2}}.$$

The sign of the numerator is positive. The sign of the denominator is not easy to see. After calculating the derivatives, we can see that the sign of the denominator equals to $Sign\left(\theta^2\left(1+\frac{R\sigma_k^2}{2(1+R\sigma_k^2)}\right)-r\right)$.

To determine the sign of the abovementioned equation, we need to examine the range of σ_k^2 on the contour line OC. We first compare two terms, $\frac{t}{2}$ and $\frac{\text{Log}(1+Rt)}{2R}$, respectively. Noticing that$R > 0$, it is more convenient to compare the following two terms Rt and $\text{Log}(1+Rt)$, which can be obtained by multiplying $2R$ to the previous two terms. It is commonly known that $Rt > \text{Log}(1+Rt)$, for all $Rt > 0$. Thus, we have $\frac{t}{2} > \frac{\text{Log}(1+Rt)}{2R}$for any positive values of t. Furthermore, for any point on the contour line OC, we must have $\pi_O(t) = \pi_C(t)$. By comparing the two functions as in the proof of Lemma 4.4, we must have the following inequality,

$$\frac{\theta^4}{2\left(\theta^2 + \theta^2\frac{Rt}{1+Rt} + R\sigma_\varepsilon^2\right)} > \frac{\theta^4}{2(2r - \theta^2 + R\sigma_\varepsilon^2)}, \qquad (A.36)$$

for any point on the contour line OC. Examining (A.36), we immediately obtain the desired result,

$$r > \theta^2\left(1 + \frac{R\sigma_k^2}{2(1+R\sigma_k^2)}\right). \qquad (A.37)$$

Thus, $\frac{d\sigma_\varepsilon^2}{d\sigma_k^2} < 0$. Notice that condition (A.37) guarantees the existence of contour line OC. If violated, the contour line OC vanishes and the outcome control is always dominated by clan control.

Proof of Theorem 4.3:

To check the curvature of the contour line OC, we take total differentiation with respect to equation (A50),[61]

$$\pi_{(0,2)}^{OUTCOME}(\sigma_\varepsilon^2,\sigma_k^2) + \frac{d^2\sigma_\varepsilon^2}{d(\sigma_k^2)^2}\pi_{(1,0)}^{OUTCOME}(\sigma_\varepsilon^2,\sigma_k^2) + \frac{d\sigma_\varepsilon^2}{d\sigma_k^2}\pi_{(1,1)}^{OUTCOME}(\sigma_\varepsilon^2,\sigma_k^2)$$

$$+ \frac{d\sigma_\varepsilon^2}{d\sigma_k^2}\left[\pi_{(1,1)}^{OUTCOME}(\sigma_\varepsilon^2,\sigma_k^2) + \frac{d\sigma_\varepsilon^2}{d\sigma_k^2}\pi_{(2,0)}^{OUTCOME}(\sigma_\varepsilon^2,\sigma_k^2)\right]$$

$$= \pi_{(0,2)}^{CLAN}(\sigma_\varepsilon^2,\sigma_k^2) + \frac{d^2\sigma_\varepsilon^2}{d(\sigma_k^2)^2}\pi_{(1,0)}^{CLAN}(\sigma_\varepsilon^2,\sigma_k^2) + \frac{d\sigma_\varepsilon^2}{d\sigma_k^2}\pi_{(1,1)}^{CLAN}(\sigma_\varepsilon^2,\sigma_k^2)$$

$$+ \frac{d\sigma_\varepsilon^2}{d\sigma_k^2}\left[\pi_{(1,1)}^{CLAN}(\sigma_\varepsilon^2,\sigma_k^2) + \frac{d\sigma_\varepsilon^2}{d\sigma_k^2}\pi_{(2,0)}^{CLAN}(\sigma_\varepsilon^2,\sigma_k^2)\right].$$

Then we solve for the second derivative as follows,

$$\frac{d^2\sigma_\varepsilon^2}{d(\sigma_k^2)^2} = \frac{\pi_{(0,2)}^{CLAN}(\sigma_\varepsilon^2,\sigma_k^2) - \pi_{(0,2)}^{OUTCOME}(\sigma_\varepsilon^2,\sigma_k^2) + 2\frac{d\sigma_\varepsilon^2}{d\sigma_k^2}[\pi_{(1,1)}^{CLAN}(\sigma_\varepsilon^2,\sigma_k^2) - \pi_{(1,1)}^{OUTCOME}(\sigma_\varepsilon^2,\sigma_k^2)] + [\frac{d\sigma_\varepsilon^2}{d\sigma_k^2}]^2[\pi_{(2,0)}^{CLAN}(\sigma_\varepsilon^2,\sigma_k^2) - \pi_{(2,0)}^{OUTCOME}(\sigma_\varepsilon^2,\sigma_k^2)]}{\pi_{(1,0)}^{OUTCOME}(\sigma_\varepsilon^2,\sigma_k^2) - \pi_{(1,0)}^{CLAN}(\sigma_\varepsilon^2,\sigma_k^2)}.$$

It is straightforward to check that $\pi_{(1,1)}^{CLAN}(\sigma_\varepsilon^2,\sigma_k^2) = 0$, $\pi_{(0,2)}^{CLAN}(\sigma_\varepsilon^2,\sigma_k^2) = 0$, $\pi_{(1,1)}^{OUTCOME}(\sigma_\varepsilon^2,\sigma_k^2) < 0$, $\pi_{(0,2)}^{OUTCOME}(\sigma_\varepsilon^2,\sigma_k^2) > 0$. Furthermore, we have

[61] We use the subscript to stand for the partial differentiation.

$$\pi_{(2,0)}^{CLAN}(\sigma_\varepsilon^2,\sigma_k^2) - \pi_{(2,0)}^{OUTCOME}(\sigma_\varepsilon^2,\sigma_k^2)$$

$$= R^2\theta^4[(\frac{1}{R\sigma_k^2 + 2r - \theta^2})^3 - (\frac{1}{R\sigma_k^2 + \theta^2 + \frac{\theta^2 R\sigma_k^2}{1+R\sigma_k^2}})] < 0,$$

after comparing the terms in the bracket and apply condition A(54). From the proof of Theorem 4.2, we already know that $\pi_{(1,0)}^{BEHAVIOR}(\sigma_\varepsilon^2,\sigma_k^2) - \pi_{(1,0)}^{CLAN}(\sigma_\varepsilon^2,\sigma_k^2) < 0$. Thus, combining the above results, we have

$$\frac{\partial^2\sigma_\varepsilon^2}{\partial^2\sigma_k^2} > 0.$$

Therefore, the contour line OC is convex and apparently the bottom right cell is more clan control than outcome control.

Proof of Corollary 4.2:

From the proof of Theorem 2, we have

$$\frac{\partial\sigma_\varepsilon^2}{\partial\sigma_k^2} = \left(\frac{\partial\pi^{CLAN}(\sigma_\varepsilon^2,\sigma_k^2)}{\partial\sigma_k^2} - \frac{\partial\pi^{OUTCOME}(\sigma_\varepsilon^2,\sigma_k^2)}{\partial\sigma_k^2}\right) \Bigg/ \frac{\partial\pi^{OUTCOME}(\sigma_\varepsilon^2,\sigma_k^2)}{\partial\sigma_\varepsilon^2} - \frac{\partial\pi^{CLAN}(\sigma_\varepsilon^2,\sigma_k^2)}{\partial\sigma_\varepsilon^2}.$$

The numerator is

$$\frac{1}{2} - \frac{1}{2(1+Rt)} + \frac{\theta^6 R}{2\left(\theta^2 + \theta^2\frac{Rt}{1+Rt} + R\sigma_\varepsilon^2\right)^2(1+Rt)^2},$$

and the denominator is

$$\frac{R\theta^4[2(1+R\sigma_k^2)(r+R\sigma_\varepsilon^2) + r\theta^2\sigma_k^2]\left[\theta^2\left(1+\frac{R\sigma_k^2}{2(1+R\sigma_k^2)}\right) - r\right]}{(1+R\sigma_k^2)(2r+R\sigma_\varepsilon^2-\theta^2)^2[\theta^2 + R(\sigma_\varepsilon^2 + \frac{\theta^2\sigma_k^2}{1+R\sigma_k^2})]^2}.$$

After applying the L'Hospital's Rule, we have

$$\lim_{R\to 0}\frac{\partial\sigma_\varepsilon^2}{\partial\sigma_k^2}=\lim_{R\to 0}\frac{\dfrac{1}{2}-\dfrac{1}{2(1+Rt)}+\dfrac{\theta^6 R}{2\left(\theta^2+\theta^2\dfrac{Rt}{1+Rt}+R\sigma_\varepsilon^2\right)^2(1+Rt)^2}}{\dfrac{R\theta^4[2(1+R\sigma_k^2)(r+R\sigma_\varepsilon^2)+r\theta^2\sigma_k^2]\left[\theta^2\left(1+\dfrac{R\sigma_k^2}{2(1+R\sigma_k^2)}\right)-r\right]}{(1+R\sigma_k^2)(2r+R\sigma_\varepsilon^2-\theta^2)^2[\theta^2+R(\sigma_\varepsilon^2+\dfrac{\theta^2\sigma_k^2}{1+R\sigma_k^2})]^2}}$$

$$=\frac{(\theta^2+\sigma_k^2)(\theta^2-2r)^2}{4r(\theta^2-r)}.$$

Condition (A.37) guarantees that the above term is always negative.